



Portofolio Management with Markowitz Model to Determine Optimal Investment Values

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Abstract

Investment is a commitment to a number of funds or other resources made at this time with the aim of obtaining a number of benefits in the future. Investments can be channeled through several instruments, such as deposits, gold, property, stocks, and many more. With the various types of investments that exist today, the difficulty of finding an investment cannot be an obstacle. In investment, the appropriate allocation of funds can be an important factor in obtaining profits. By using the correct method, the risk factors that may occur can be minimized as well as possible. The method that can be used in determining funds in investment activities is using the Markowitz Model. Then, the author initiates a method for optimizing assets using the Markowitz method. In this research, used data from 10 stocks in Indonesia PANI, CLEO, DSSA, UNIC, ADRO, CITA, CAKK, TPIA, MYOR, ANTM. Then, the stocks will be arranged optimally portfolio. The optimal investment weight obtained for stocks from January 1, 2019 to December 31, 2023 using the Markowitz model, each share weighting namely 7,6% PANI, 18,05% CLEO, 8,48% DSSA, 16,902% UNIC, 12,471% ADRO, 10,496% CITA, 7,246% CAKK, 12,946% TPIA, 5,877% MYOR, and 0,464% ANTM company stocks and provide a portfolio ratio of 5.694581 which can also be interpreted that the optimal return ratio profit is 5.694581 times greater than the possible loss or portfolio variance.

Keywords: Markowitz Model, Stock, Optimal Invesment, Portofolio

1. Introduction

In this modern era, all activities can be done easily, one of which is investment activity. Investment is a commitment to a number of funds or other resources made at this time with the aim of obtaining a number of benefits in the future Investments can be channeled through several instruments, such as deposits, gold, property, stocks, and many more (Bodie et al., 2013). With the various types of investments that exist today, the difficulty of finding an investment cannot be an obstacle.

Investments with profits that are easily disbursed, can be done quickly, and are easy to succeed are popular targets for investors today. One of the investment objects is investment in stocks. With the development of technology, stock investment can be done anywhere as long as it is within the reach of the internet, either using a computer device, or using gadgets that are always used daily (Fleisch, 2010).

Investing in stocks is usually done through intermediaries, either by going directly to the securities office that has obtained an official license to trade stocks or by investing online through an official application under the supervision of the Financial Services Authority in Indonesia. With the many choices of intermediaries and all the conveniences that exist to invest in stocks today, making a profit is still one of the things that cannot be missed.

In investment, the appropriate allocation of funds can be an important factor in obtaining profits (Azwari et al., 2022). By using the correct method, the risk factors that may occur can be minimized as well as possible. The method that can be used in determining funds in investment activities is using the Markowitz Model (Yuwono & Ramdhani, 2017). In this study, the Markowitz Model will be used to allocate funds or investment weights that aim to obtain optimal investment portfolio management results.

2. Literature Review

2.1. Investment

Investment is the placement of money or property in the hope that the money or property can increase in value in the future (Baum & Hartzell, 2012). According to Fabozzi & Markowitz (2011) there are 5 stages in determining investment decisions including: Determining investment objectives, determining investment policy, selecting portfolio strategies, selecting assets, and measuring & evaluating portfolio performance. Based on the above definition, it can be concluded that investment is a form of wealth sacrifice in the present to get benefits in the future with a certain level of risk.

2.2. Stock

Stocks are a sign of ownership of a company that represents management to run the company (Xu & Wang, 1999). A stockholder has voting rights in the general meeting of stockholders and in addition to obtaining dividends from the company, a stockholder is also likely to benefit from the increase in stock prices (Easterbrook & Fischel, 1983).

2.3. Return

The sum of yield and capital gains is called total return of an investment (Melichar, 1979). Yield is a component of return that reflects the cash flow or income earned periodically from an investment. Capital gain is an increase in the price of something valuable (stocks or long-term debt securities) that can provide profits for investors.

$$R_{it} = \frac{P_{it} - P_{it-1}}{P_{it-1}} \quad (1)$$

where,

R_{it} : stock return i in period t

P_{it} : stock price i in period t

2.4. Variance-Covariance Matrix

The variance-covariance matrix is a symmetric matrix that shows the variance of each variable and the covariance between each pair of variables in a dataset. In statistics and probability, this matrix is very important as it provides information on how much the variables vary and how they are related to each other.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \dots & \sigma_{1,N} \\ \sigma_{2,1} & \sigma_2^2 & \dots & \sigma_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N,1} & \sigma_{N,2} & \dots & \sigma_N^2 \end{bmatrix} \quad (2)$$

where,

σ : covariance of the return

σ^2 : variance of the return

2.5. Lagrange Multiplier

Lagrange multiplier is a method in optimization used to find the extremum point (maximum or minimum) of a function with constraint conditions. This method introduces an additional multiplier called as Lagrange multiplier to transform a constrained optimization problem into an unconstrained optimization problem.

$$\lambda = \frac{e^T \Sigma^{-1} \mu - \frac{\rho}{2}}{e^T \Sigma^{-1} e} \quad (3)$$

where,

e : unit vector

Σ : covariance variance matrix

ρ : risk aversion

μ : average return

2.6. Markowitz Portfolio

Portfolio management is a process of optimally managing assets in order to generate maximum return as well as minimal risk, this is an important application in finance. The foundation of modern portfolio theory was introduced by Harry Markowitz in a paper entitled "Portfolio Selection" in 1952, which introduced us to the concept of Mean-Variance Analysis. The Markowitz model is a basic model for optimizing asset weights in an investment portfolio over a single period with the primary objective of maximizing the mean return and minimizing the variance of the return. The Markowitz model provides a systematic approach to calculating the expected return and risk for different assets, helping investors to make more rational and informed investment decisions.

$$w = \frac{2}{\rho} \Sigma^{-1} (\mu - \lambda e) \quad (4)$$

$$\mu_p = \mu^T w \quad (5)$$

$$Var_p = w^T \Sigma w \quad (6)$$

$$Ratio = \frac{\mu_p}{Var_p} \quad (7)$$

where,

w : portfolio value
 μ : average return
 λ : lagrange multiplier
 e : unit vector
 Σ : covariance variance matrix
 ρ : risk aversion
 μ_p : return of portfolio
 Var_p : variance of the portfolio

3. Materials and Methods

3.1. Materials

This study will use secondary data with the type of time series data in the January 2019-December 2023 period. The data used in this study is data derived from the closing stock index of 10 domestic companies, namely PANI, CLEO, DSSA, UNIC, ADRO, CITA, CAKK, TPIA, MYOR, ANTM. Data obtained from the website www.finance.yahoo.com. This data will be processed using the Microsoft Excel application.

3.2. Methods

The Markowitz model is a basic model for optimizing asset weights in an investment portfolio in a single period with the main objective of maximizing the average return and minimizing the variance of the return. The following are the steps in the application of the Markowitz Model.

- (a) Find the closing price of the stock of each company that will be selected for the study.
- (b) Calculate the stock return from the closing price of the stock that has been obtained previously.
- (c) Calculate the average return of each closing price stock return.
- (d) Determine the unit vector to use.
- (e) Find the variance covariance matrix (Σ) from the results of all stock returns previously obtained.
- (f) Determine the initial risk aversion amount to determine the investment value.
- (g) Calculate the lagrange multiplier (λ) using the third equation formula.
- (h) Calculate the portfolio value (vector w) using the fourth equation formula.
- (i) Calculate the portfolio return.
- (j) Calculate the variance of the portfolio.
- (k) Calculate the Ratio of the portfolio.

- (l) Determine the amount of risk aversion again to see the best portfolio ratio so that the optimal investment value is obtained.

4. Results and Discussion

The results of this study include the process of determining the optimal portfolio value of 10 stocks in Indonesia in the January 2019-December 2023 period using the Markowitz method. The first step in the Markowitz method is to find the closing price of the shares of the previously selected company. Then in the next step, the return value of the company's previous closing stock will be sought.

Table 1: Stocks price table

Stocks Price											
No	Date	PANI	CLEO	DSSA	UNIC	ADRO	CITA	CAKK	TPIA	MYOR	ANTM
1	01/01/2019	11,580595	282	15200	4640	1390	1650	99	1468,75	2580	965
2	01/02/2019	7,601986	280	15525	4420	1310	1500	145	1425	2640	1015
3	01/03/2019	8,383498	276	14500	5125	1340	1400	256	1387,5	2570	885
4	01/04/2019	8,099312	322	14500	5525	1305	1350	120	1275	2600	865
5	01/05/2019	7,530939	380	14500	4180	1295	1500	108	1275	2560	725
6	01/06/2019	8,028265	388	14500	4500	1360	1600	112	1242,5	2480	845
7	01/07/2019	8,028265	525	16950	5225	1270	1795	120	1568,75	2490	935
8	01/08/2019	8,170359	590	19975	4260	1125	1990	112	2206,25	2490	1070
9	01/09/2019	7,388846	540	19175	3800	1290	1920	107	2050	2230	975
:	:	:	:	:	:	:	:	:	:	:	:
59	01/11/2023	4700	655	51500	8025	2620	2070	160	2950	2550	1740
60	01/12/2023	4900	710	80000	8200	2380	2110	185	5250	2490	1705

Table 2: Stocks return table

Stocks Return											
No	Date	PANI	CLEO	DSSA	UNIC	ADRO	CITA	CAKK	TPIA	MYOR	ANTM
1	01/02/2019	-0.344	-0.007	0.0214	-0.047	-0.058	-0.091	0.4646	-0.03	0.0233	0.0518
2	01/03/2019	0.1028	-0.014	-0.066	0.1595	0.0229	-0.067	0.7655	-0.026	-0.027	-0.128
3	01/04/2019	-0.034	0.1667	0	0.078	-0.026	-0.036	-0.531	-0.081	0.0117	-0.023
4	01/05/2019	-0.07	0.1801	0	-0.243	-0.008	0.1111	-0.1	0	-0.015	-0.162
5	01/06/2019	0.066	0.0211	0	0.0766	0.0502	0.0667	0.037	-0.025	-0.031	0.1655
6	01/07/2019	0	0.3531	0.169	0.1611	-0.066	0.1219	0.0714	0.2626	0.004	0.1065
7	01/08/2019	0.0177	0.1238	0.1785	-0.185	-0.114	0.1086	-0.067	0.4064	0	0.1444
8	01/09/2019	-0.096	-0.085	-0.04	-0.108	0.1467	-0.035	-0.045	-0.071	-0.104	-0.089
9	01/10/2019	-0.058	0.0093	-0.087	0	0.0155	0.0026	-0.103	0.1585	-0.04	-0.087
:	:	:	:	:	:	:	:	:	:	:	:
58	01/11/2023	0.0755	0.0234	0.0098	-0.006	0.0234	-0.037	-0.006	-0.01	-0.105	0.0205
59	01/12/2023	0.0426	0.084	0.5534	0.0218	-0.092	0.0193	0.1563	0.7797	-0.024	-0.02

In the third step, the average return of each company's stock will be calculated, then create a unit vector with 10 rows according to the number of companies.

Table 3: Average stock return table

Stocks	Average of Return
PANI	0.196534553
CLEO	0.020738932
DSSA	0.04268601
UNIC	0.016122561
ADRO	0.017271966
CITA	0.008019327
CAKK	0.034445644
TPIA	0.035361975
MYOR	0.002140666
ANTM	0.021468646

The next step, find the variance covariance matrix from the results of all stock returns that were previously obtained.

Table 4: Variance covariance matrix table

VARIANCE COVARIANCE MATRIX										
	PANI	CLEO	DSSA	UNIC	ADRO	CITA	CAKK	TPIA	MYOR	ANTM
PANI	0.4272	-0.002	0.0523	-0.004	0.0033	-0.004	0.0193	-0.002	-0.006	0.011
CLEO	-0.002	0.011	0.0033	0.0018	0.0014	0.001	0.0003	0.0054	0.0033	0.0031
DSSA	0.0523	0.0033	0.034	0.0021	-0.004	-6E-04	-0.003	0.0058	-0.001	-0.002
UNIC	-0.004	0.0018	0.0021	0.014	0.0012	0.0019	-0.004	0.0009	0.0001	0.0057
ADRO	0.0033	0.0014	-0.004	0.0012	0.0164	0.0016	0.0079	-6E-04	-9E-04	0.0027
CITA	-0.004	0.001	-6E-04	0.0019	0.0016	0.0081	-0.003	0.0027	0.0004	0.0052
CAKK	0.0193	0.0003	-0.003	-0.004	0.0079	-0.003	0.061	0.0033	-0.001	0.0017
TPIA	-0.002	0.0054	0.0058	0.0009	-6E-04	0.0027	0.0033	0.0326	0.0008	0.004
MYOR	-0.006	0.0033	-0.001	0.0001	-9E-04	0.0004	-0.001	0.0008	0.0058	0.0006
ANTM	0.011	0.0031	-0.002	0.0057	0.0027	0.0052	0.0017	0.004	0.0006	0.0263

Choose 11 as the initial risk aversion amount to determine the portfolio weight. Then a lagrange multiplier of 0.000421 is obtained and the initial portfolio weights to be used are as follows.

Table 5: Portfolio value table

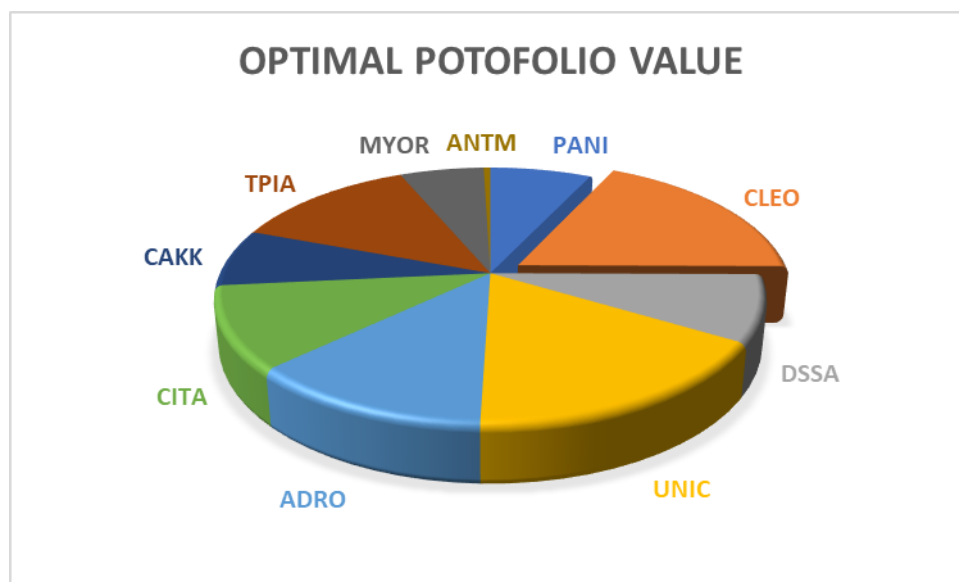
Stocks	Value
PANI	0.073116
CLEO	0.187395
DSSA	0.084714
UNIC	0.171015
ADRO	0.124685
CITA	0.101009
CAKK	0.073333
TPIA	0.133322
MYOR	0.046461
ANTM	0.004949

To find the final result of the optimal portfolio value, first find the portfolio return value (Eq 4) and the portfolio variance value (Eq 5). After obtaining the portfolio return value and portfolio variance value, find the portfolio ratio value by dividing the results of the portfolio return value by the portfolio variance value.

Then choose the final risk aversion value of 11.38916 so as to get the maximum portfolio ratio value of 5.694581. Then get the optimal portfolio weight value as follows.

Table 6: Optimal portfolio value table

Stocks	Value
PANI	0.070619
CLEO	0.180521
DSSA	0.084807
UNIC	0.169024
ADRO	0.124719
CITA	0.104964
CAKK	0.072463
TPIA	0.129462
MYOR	0.058777
ANTM	0.004644

**Figure 1:** Optimal portfolio value pie chart

5. Conclusion

Based on the results and discussion that have been obtained, it is known that the optimal investment weight for stocks from January 1, 2019 to December 31, 2023 using the Markowitz model, each share weighting namely 7.06% PANI, 18.05% CLEO, 8.48% DSSA, 16.902% UNIC, 12.471% ADRO, 10.496% CITA, 7.246% CAKK, 12.946% TPIA, 5.877% MYOR, and 0.464% ANTM company stocks. From the optimal portfolio formed, it is also known to provide a portfolio ratio of 5.694581 which can also be interpreted that the optimal return ratio profit is 5.694581 times greater than the possible loss or portfolio variance.

References

- Azwari, P. C., Febriansyah, F., & Jayanti, S. D. (2022). Impact of Third-Party Funds and Capital Adequacy Ratio on Profit Sharing Financing. *International Business and Accounting Research Journal*, 6(1), 63-70.
- Baum, A. E., & Hartzell, D. (2012). *Global property investment: Strategies, structures, decisions*. John Wiley & Sons.
- Bodie, Z., Kane, A., & Marcus, A. (2013). *Ebook: Essentials of investments: Global edition*. McGraw Hill.
- Easterbrook, F. H., & Fischel, D. R. (1983). Voting in corporate law. *The journal of Law and Economics*, 26(2), 395-427.
- Fabozzi, F. J., & Markowitz, H. M. (Eds.). (2011). *The theory and practice of investment management: Asset allocation, valuation, portfolio construction, and strategies* (Vol. 198). John Wiley & Sons.

- Fleisch, E. (2010). What is the internet of things? An economic perspective. *Economics, Management, and financial markets*, 5(2), 125-157.
- Melichar, E. (1979). Capital gains versus current income in the farming sector. *American Journal of Agricultural Economics*, 61(5), 1085-1092.
- Xu, X., & Wang, Y. (1999). Ownership structure and corporate governance in Chinese stock companies. *China economic review*, 10(1), 75-98.
- Yuwono, T., & Ramdhani, D. (2017). Comparison analysis of portfolio using Markowitz model and single index model: Case in Jakarta Islamic Index. *Journal of Multidisciplinary Academic*, 1(1), 25-31.