



Calculation of Life Insurance Premiums with Markov Chain Applications for Patients with Pulmonary Tuberculosis Disease in Indonesia

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Abstract

This study aims to calculate the premium for Long Term Care (LTC) insurance on Annuity as A Rider Benefit products with a multi-status model. Pulmonary Tuberculosis is one of the major infectious diseases that contribute significantly to morbidity and mortality rates. Therefore, it is necessary to calculate insurance premiums that consider the health risks of sufferers. In this study, Markov chains are used to model the health status transition of individuals with Pulmonary Tuberculosis over time, considering several health states, such as healthy, sick, and dead. Tuberculosis epidemiological data in Indonesia is used to estimate the transition probabilities between health states. The case study used in this research is a 35-year-old man who participates in LTC insurance with a coverage period of 5 years. It is known that the value of compensation when a person dies is IDR 100,000,000. The interest rate used is 5%. The calculation results obtained annual premium for LTC insurance on Annuity as A Rider Benefit product is Rp294,333. Then the calculation of the annual net premium of this insurance is also calculated based on age and gender. When age increases, the premium will be greater, this is influenced by the greater chance of death when age increases. In addition, based on gender, it is found that the male premium is more expensive than the female gender. This is influenced by the chance of death of men is greater than women.

Keywords: Life Insurance, Annuity as A Rider Benefit, Markov Chain

1. Introduction

Tuberculosis (TB) is one of the most pressing public health problems in Indonesia. According to data from the Ministry of Health, Indonesia is one of the countries with the highest TB burden in the world. The disease is caused by *Mycobacterium tuberculosis* and can be transmitted through the air, thus requiring special attention in its management. In addition to the health impact, TB also places a heavy economic burden on individuals and families, especially in terms of prolonged treatment costs.

Insurance is an agreement in which the insurer binds itself to the insured with a promise to pay the insured to compensate for losses caused by loss, damage, or inability to obtain expected profits due to unknown events (Winter, 1991). The main purpose of insurance is to transfer risk from the insured to the insurer. This process does not eliminate the possibility of an adverse event, but provides financial protection and a sense of security for the insured. One important aspect of insurance is the payment that the insured makes to the insurer as compensation for the protection provided (Konetzka, 2014). Term life insurance provides life insurance protection for a certain period of time, with benefits to be paid if an unexpected event occurs. The advantage of term life insurance is the flexibility to choose how long the insured party wants. Prospective insureds usually need this term life insurance because they need temporary protection and have a small income but need significant protection at a low cost (Pitacco, 2014). In this context, life insurance is important to protect individuals from the financial risks posed by the treatment of this disease. Life insurance can help ease the burden of costs that must be borne by patients and their families. However, to determine the right insurance premium, an accurate and data-driven approach is required. One method that can be used is mathematical modeling, specifically the application of Markov chains.

Markov chains are a statistical technique that allows the analysis of transitions between various patient health states. In the case of tuberculosis patients, health status can be grouped into several categories, such as healthy, pulmonary TB, extra-pulmonary TB, and dead. Using this model, we can calculate the transition probabilities between states based on available epidemiological data. This provides a clearer picture of the risks faced by people with TB. The use of Markov chain model in life insurance premium calculation has several advantages. First, the model can

provide a more realistic estimate of health risk based on the transition patterns that occur in patients. Second, by considering factors such as age, gender, and other health conditions, insurance companies can determine premium rates that are more fair and in accordance with the risk profile of each individual.

Various studies have been conducted to develop mathematical finance, especially in the context of healthcare. Private LTC insurance products must be innovative to meet the growing demand, emphasizing the need for effective risk pooling among heterogeneous individuals (Kabuche et al., 2024). It is noted that LTC spending in the US and other countries has increased significantly, with most remaining uninsured due to declining informal care and insufficient public financing. Long-Term Care (LTC) was highlighted as an important focus area for healthcare institutions, especially for individuals who are not independent due to illness (Mucha et al., 2022). The financial implications of LTC are significant, prompting consideration for long-term care insurance to mitigate rising healthcare costs. Previous studies on critical illness insurance, particularly in relation to diseases such as Alzheimer's and breast cancer, were reviewed to build a foundation for the current study on breast cancer using a multi-state model. (Fathoni et al., 2022). It highlights the importance of Markov models in understanding transitions between states such as health, illness, and death, emphasizing their application in critical illness insurance research. Various mathematical models have been developed to assess cyber risk, including non-network and network models, with a focus on the impact of network structure on cyber insurance pricing (Antonio et al., 2021). This article highlights the growing need for cyber risk management through cyber insurance, especially as cyber threats evolve and increase every year.

This study aims to develop a multi-status model for LTC insurance and determine the Markov Chain transition matrix to model the transition between health states. Using data on the prevalence of pulmonary tuberculosis disease in Indonesia in 2023, this study will calculate premiums for LTC insurance products, especially Annuity as A Rider Benefit. The calculation process involves linear interpolation of prevalence data, construction of a transition matrix, and determination of the insured's age, coverage time, and interest rate. The results of this study will provide an annual premium value that can be used as a guide in paying insurance premiums.

Table 1: Research gap analysis

Author	Variable	Method	Markov chain usage	Linear interpolation usage	Net premium calculation
Mucha et al., 2022	Long-term care delivery	Markov chain, monte carlo simulation	Yes	No	No
Fathoni et al., 2022	Chest cancer patients	Markov process, Kolmogorov process	Yes	No	No
Antonio et al., 2021	Cyber insurance premium	Markov Model	Yes	No	Yes
Kabuche et al., 2024	Functional disability and health status	Markov chain	Yes	No	No
Esquivel et al., 2024	Simulation data	Markov chain, Gompertz-Makeham transition	Yes	No	No
Cui et al., 2022	Chinese public health survey data	Markov chain	Yes	No	Yes
Spreeuw et al., 2022	Transition from healthy state to next state	Markov chain	Yes	No	No
This research	Patients with pulmonary tuberculosis	Markov chain	Yes	Yes	Yes

2. Literature Review

2.1. Life Insurance

Life insurance is a financial contract that provides financial protection to the policyholder's beneficiaries in the event of the policyholder's death. In the context of term life insurance, this protection is only valid for a certain period. After the period ends, benefits are not provided if the policyholder is still alive (Smith, 1982).

Term life insurance is a type of insurance that provides financial protection for a certain period of time. If the policyholder dies during the insurance period, the beneficiary will receive the benefits of the insurance (Smith, 1982). If the policyholder is still alive after the expiry period, this insurance does not provide financial benefits. Therefore, the premium calculation for this product needs to consider various probability factors, such as age, health, and insurance duration by using a multi-state approach as a method to model states and transitions.

The multi-state models used in this study include:

- **Life State Model:** Identifies the various states that individuals may experience, including healthy states, states with diseases, and states of death.
- **Transitions Between States:** Takes into account the probability of transitioning from one state to another within a certain period of time.
- **Effect of Policy Duration:** Considers how the policy duration affects the probability and risk of death, as well as its effect on the premium calculation.

2.2. Annuity as a Rider Benefit

An annuity as a rider benefit is an additional feature that can be included in life insurance products to provide regular income to policyholders. This rider is often used to supplement the basic benefits of the policy by providing a regular income stream, especially in the context of retirement planning or long-term financial protection (Almeida et al., 2009).

Annuities are financial contracts that provide periodic payments to individuals over a period of time, usually throughout their lives. The main purpose of annuities is to provide a stable income guarantee, especially during retirement.

Rider is an addition to an insurance policy that provides extra benefits beyond the basic protection. Riders are often used to customize policies to the specific needs of policyholders, such as additional protection or financial benefits.

In the context of life insurance, annuities can be considered as riders that offer periodic payment benefits after the policyholder reaches retirement age or under certain conditions. This rider provides additional benefits that can complement life insurance protection with guaranteed income. Its application in the context of Long Term Care (LTC) Insurance includes care funding, financial protection, and risk management (Konetzka, 2014).

2.3. Long-term Care Insurance

Long-term care (LTC) insurance is an insurance product designed to help cover the costs of prolonged and often expensive care, such as home care, nursing home care, or home medical care (Konetzka, 2014). LTC insurance is a type of insurance that provides financial protection for long-term care costs that are not covered by traditional health insurance or Medicare. This includes a variety of services such as home care, nursing home care, and other health care services. Risk calculation Insurance companies use epidemiological and actuarial data to calculate risks and set premiums. This involves analyzing the probability of long-term care needs and estimating the associated costs.

In the article by Almida et al. (2009) a multistate model is applied to calculate LTC insurance premiums by considering the specific health status and transition probabilities relevant for a given population. This involves:

- **Health Data:** Collecting data on transition probabilities between health states for the population under consideration.
- **Risk Estimation:** Using the model to estimate the risk faced by policyholders and calculating premiums based on that risk.

2.4. Markov Chain

Markov chain is a mathematical model used to analyze systems that experience random status changes. In the context of calculating life insurance premiums, Markov chains are very useful for predicting the transition opportunities between healthy, sick, and dead statuses (Kausch et al., 2021).

The transition probability matrix is used to determine the probability of changing from one status to another. By using data on the prevalence rate of Lung TB disease and the transition opportunity matrix, it shows the importance of mathematical models in the insurance field to minimize risk and increase customer financial security.

3. Materials and Methods

3.1. Materials

The materials used in this study consisted of epidemiological data related to Lung TB disease in Indonesia. The data used was obtained from the Ministry of Health of the Republic of Indonesia (2023) and other relevant sources. This data includes the number of pulmonary tuberculosis cases per year in Indonesia, cure and death rates due to pulmonary tuberculosis, demographic data such as age, gender, and risk factors associated with pulmonary tuberculosis. This data will be used to build a transition matrix that describes changes in the health status of patients from healthy to sick with Lung TB and finally death.

3.2. Methods

The following are the methods carried out in this research:

- (a) Indonesian Health Survey (IHS) data related to pulmonary tuberculosis patients in Indonesia will be taken from the website www.kemkes.go.id.
- (b) Linear interpolation of the prevalence rate of pulmonary tuberculosis in Indonesia will be calculated.

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \quad (1)$$

- (c) Next, the transition matrix will be compiled.

$${}_h P_x = \begin{bmatrix} p_x^{11} & p_x^{12} & q_x^{13} \\ 0 & p_x^{22} & q_x^{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_{x+1}^{11} & p_{x+1}^{12} & q_{x+1}^{13} \\ 0 & p_{x+1}^{22} & q_{x+1}^{23} \\ 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{x+h-1}^{11} & p_{x+h-1}^{12} & q_{x+h-1}^{13} \\ 0 & p_{x+h-1}^{22} & q_{x+h-1}^{23} \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

with:

$$\begin{aligned} p_x^{11} &= 1 - p_x^{12} - q_x^{13} \\ q_x^{23} &= (1 + \eta) \times q_x^{13} \\ p_x^{22} &= 1 - q_x^{23} \end{aligned} \quad , \eta \geq 0$$

- (d) Calculate the net premium of LTC insurance

The net single premium value of LTC insurance is

$$A_{x:n}^{LTC} = c \sum_{e=1}^n v^e {}_e p_x^{11} q_{x+e-1}^{13} + b \sum_{e=1}^n [v^e {}_e p_x^{11} q_{x+e-1}^{12} (\ddot{a}_{x+e:r}^{22} + \sum_{f=1}^c (c - fb) v^f {}_{f-1} p_{x+e}^{22} q_{x+e+f-1}^{23})] \quad (3)$$

The insurance premium value is

$$P = \frac{A_{x:n}^{LTC}}{a_{x:n}^{ij}} \quad (4)$$

with:

P : The value of the insurance premium that must be paid by the insured.

$A_{x:n}^{LTC}$: The net single premium value of LTC insurance that has been calculated in the previous formula.

$a_{x:n}^{ij}$: The cash value of premium payments made at regular intervals, used to divide a single premium value into smaller, recurring premium payments.

4. Results and Discussion

The data on the prevalence rate of pulmonary tuberculosis used is Indonesian basic health research data in 2023.

Table 2: Percentage of Patients with Pulmonary TB in Indonesia

Age group (years)	Prevalence rate (%)
<1	0.08
1-4	0.42
5-14	0.18
15-24	0.18
25-34	0.26
35-44	0.28
45-54	0.37
55-64	0.51
65-74	0.59
75+	0.50
Male	0.38
Female	0.22
Average	0.3

Based on information from Table 2, data on the prevalence of pulmonary tuberculosis disease in Indonesia in 2023 has an average of 0.3% of people experiencing pulmonary tuberculosis disease. The percentage of Lung TB disease in Indonesia, including men, is 0.38%, while women are 0.22%.

Based on data on the prevalence of Lung TB disease sufferers in Indonesia, the value of interpolation results will be determined with each age of Lung TB disease sufferers using the interpolation formula.

Suppose you want to know the prevalence of people with Lung TB disease (y) of a person at the age of (x) 2 years, then the calculation is

$$x = 2 \text{ year} \quad x_1 = 1 \text{ year} \quad x_2 = 5 \text{ year}$$

y : The result of the prevalence rate of people with Lung TB disease at age 2 years.

y_1 : The prevalence rate of Lung TB disease at age 1.

y_2 : Prevalence rate of Lung TB disease at age 5 years.

$$\begin{aligned}
 y &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \\
 &= \frac{0.18 - 0.42}{5 - 1}(2 - 1) + 0.42 = 0.36
 \end{aligned}$$

The calculation is done in the same way for the next age level, resulting in the interpolated results in Table 3.

Table 3: Results of Linear Interpolation of Lung TB

Age	Prevalance rate
1	0.08
2	0.36
3	0.3
4	0.24
5	0.18
25	0.26
35	0.28
45	0.37
55	0.51
75	0.5

Preparation of Transition Matrix

Based on the results of the prevalence rate and the Indonesian Mortality Table (TMI) in 2019, it is possible to prepare a transition matrix at each age. This transition matrix is used to determine the opportunity value of the status at the current time to the status at another time. In this research case, it is assumed to be one-way, for example, the transition from healthy status to sick status, but there is no transition from sick status to healthy status (no healthy state). The transition probability matrix h steps with three states, namely:

$${}_h P_x = \begin{bmatrix} p_x^{11} & p_x^{12} & q_x^{13} \\ 0 & p_x^{22} & q_x^{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_{x+1}^{11} & p_{x+1}^{12} & q_{x+1}^{13} \\ 0 & p_{x+1}^{22} & q_{x+1}^{23} \\ 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{x+h-1}^{11} & p_{x+h-1}^{12} & q_{x+h-1}^{13} \\ 0 & p_{x+h-1}^{22} & q_{x+h-1}^{23} \\ 0 & 0 & 1 \end{bmatrix}$$

with,

$$\begin{aligned} p_x^{11} &= 1 - p_x^{12} - q_x^{13} \\ q_x^{23} &= (1 + \eta) \times q_x^{13} \\ p_x^{22} &= 1 - q_x^{23} \end{aligned}, \eta \geq 0$$

Suppose a man is 35 years old, the calculation of the one-step transition probability matrix of a 35-year-old man is:

(a) The probability of death (q_x^{13}) of a 35-year-old one year later, $q_{35}^{13} = 0.00107$ this value is obtained from the 35-year-old mortality table.

(b) The probability of a 35-year-old man in a healthy state (1) becoming a status suffering from Lung TB disease (2) one year later, namely $p_{35}^{12} = 0.00280$

(c) The probability that a 35-year-old man in good health will remain in good health one year later is:

$$\begin{aligned} p_x^{11} &= 1 - p_x^{12} - q_x^{13} \\ p_{35}^{11} &= 1 - 0.00280 - 0.0011 \\ p_{35}^{11} &= 0.99827 \end{aligned}$$

(d) The probability of death of a 35-year-old man with pulmonary tuberculosis one year later with a constant value of $\eta = 0.05$ is:

$$\begin{aligned} q_x^{23} &= (1 + \eta) \times q_x^{13} \\ q_{35}^{23} &= (1 + 0.05) \times 0.00107 = 0.0011235 \end{aligned}$$

(e) The probability that a 35-year-old man who has Lung TB will remain sick with Lung TB one year later is:

$$\begin{aligned} p_x^{22} &= 1 - q_x^{23} \\ p_{35}^{22} &= 1 - 0.0011235 = 0.9988765 \end{aligned}$$

The following one-step transition probability matrix is obtained:

$$P_{35} = \begin{bmatrix} 0.99827 & 0.0028 & 0.00107 \\ 0 & 0.9988765 & 0.0011235 \\ 0 & 0 & 1 \end{bmatrix}$$

The transition probability matrix can be depicted in a diagram:

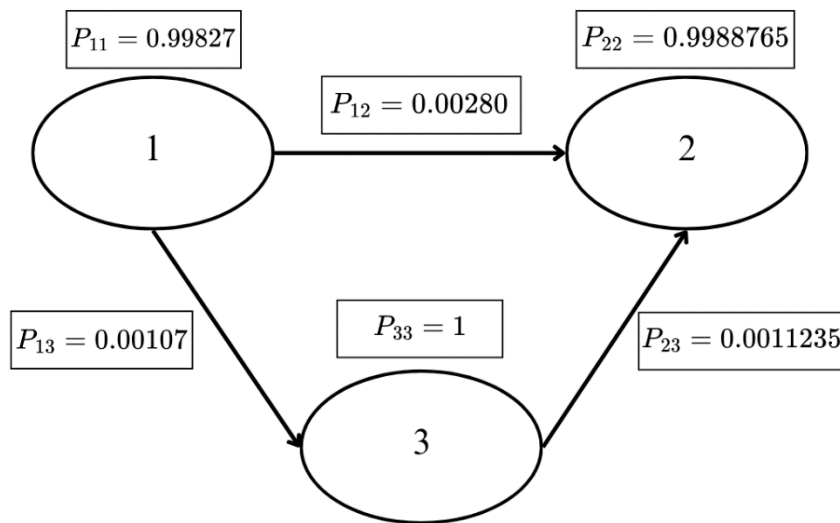


Figure 1: One-step transition diagram for a 35-year-old male

Based on the one-step transition probability matrix and Figure 3, it can be seen that the healthy probability for a 35-year-old man to be in a state of illness due to heart disease (from status 1 to status 2) before the age of 36 is 0.00280. The chance of someone dying from Lung TB before the age of 36 is 0.0011235. Furthermore, the calculation of the transition probability matrix h steps for a 35-year-old person is obtained by multiplying the matrix where $h = 1, 2, 3, 4, 5$. The matrix multiplication is taken 5 times the transition step, the 5-step transition probability matrix is as follows:

$${}_1 P_{35} = \begin{bmatrix} 0.99827 & 0.0028 & 0.00107 \\ 0 & 0.9988765 & 0.0011235 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
{}_2P_{35} &= \begin{bmatrix} 0.9965 & 0.0056 & 0.0021 \\ 0 & 0.9978 & 0.0022 \\ 0 & 0 & 1 \end{bmatrix} \\
{}_3P_{35} &= \begin{bmatrix} 0.9948 & 0.0084 & 0.0032 \\ 0 & 0.9966 & 0.0034 \\ 0 & 0 & 1 \end{bmatrix} \\
{}_4P_{35} &= \begin{bmatrix} 0.9931 & 0.0112 & 0.0043 \\ 0 & 0.9955 & 0.0045 \\ 0 & 0 & 1 \end{bmatrix} \\
{}_5P_{35} &= \begin{bmatrix} 0.9914 & 0.0139 & 0.0054 \\ 0 & 0.9944 & 0.0056 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

This Long Term Care insurance product provides benefits for individuals aged x years with the following conditions:

- Age of the Insured: At the time of the policy agreement, the insured (policyholder) is in good health.
- Death Benefit: If the insured dies at the age of x years and not due to heart disease (indicated by q_x), the beneficiary will receive a lump sum of c .
- Treatment Benefit for Heart Disease: If the insured is diagnosed with a heart disease, at the beginning of each year he/she will receive a maintenance benefit of b for a maximum of r years, where b is calculated as $b=c/r$.
- Dying Before the End of the Nursing Benefit Period: If an insured person aged x years dies before the end of the benefit period (before r years), i.e. in year h , the beneficiary will receive a benefit of $c-b \cdot \min(h, r)$.
- Premium Payment: Premiums are paid when the insured is healthy and can be in the form of a single premium or annuity paid annually.

Thus, the structure of benefits and premiums in this insurance product is designed to provide financial protection in accordance with the health condition of the insured.

In this study, an example case is taken of the calculation of the net premium value of insurance from a person who participates in LTC insurance in the Annuity as A Rider Benefit product at the age of 35, with a compensation of Rp100,000,000 if he dies. The insured also wants a number of other benefits in the form of annuity payments for a maximum of 5 years if undergoing treatment (status 2). The premium is paid at the beginning of each year for 5 years as long as the insured is in good health (status 1). The interest rate used is 5%. Thus, it is calculated the amount of annual premium that must be paid by the insured during the agreement time to get benefits in accordance with the agreement.

The calculation of the single insurance premium is as follows:

$$\begin{aligned}
A_{x:n}^{LTC} &= c \sum_{e=1}^n v^e {}_e-1 p_x^{11} q_{x+e-1}^{13} + b \sum_{e=1}^n [v^e {}_e-1 p_x^{11} q_{x+e-1}^{12} (\ddot{a}_{x+e:r}^{22} + \sum_{f=1}^{\frac{c}{b}} (c-fb) v^f {}_{f-1} p_{x+e}^{22} q_{x+e+f-1}^{23})] \\
A_{35:51}^{LTC} &= 100juta \sum_{e=1}^n \left(\frac{1}{1+0,05}\right)^e {}_e-1 p_x^{11} q_{x+e-1}^{13} + 20juta \sum_{e=1}^n \left(\frac{1}{1+0,05}\right)^e {}_e-1 p_x^{11} q_{x+e-1}^{12} \ddot{a}_{x+e:51}^{22} \\
A_{35:51}^{LTC} &= Rp1.562.252
\end{aligned}$$

The annualized net premium is obtained as follows:

$$P(A_{35:51}^{LTC}) = \frac{A_{35:51}^{LTC}}{\sum_{h=0}^5 v^h {}_h p_{35}^{11}} = \frac{Rp1.562.252}{5,3077} = Rp294.333$$

It can be seen that the calculation of the annual net premium of this insurance is also calculated with varying ages and genders. The ages used are 25, 35, 45 and 55 years old.

Tabel 4: LTC Insurance Net Premiums by Gender and Age Variance

Age	Gender	
	Male	Female
25	234.099	222.447
35	294.333	286.308
45	491.545	465.785
55	948.542	821.914

Based on Table 4, it can be concluded that as a person gets older, the premium cost will increase.

5. Conclusion

Based on the results of the discussion carried out in this study, it can be concluded that the annual net premium value that must be paid based on age and gender is: 25 years old male amounted to Rp234,099 and female amounted to Rp222,447. Age 35 years old male amounted to Rp294,333 and female amounted to Rp286,308. Age 45 years, males amounted to Rp491,545 and females amounted to Rp465,785. Meanwhile, the premium at the age of 55 years for men is Rp948,542 and for women is Rp821,914. It can be concluded that the older a person is, the more expensive the premium will be, because the chance of being affected by the chance of death or the chance of being attacked by heart disease is higher when age increases. It is also based on gender that the annual premium for men is more expensive than women because it is influenced by the chance of death or the chance of developing heart disease for a man is greater than a woman.

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