



Application of Pure Birth to the Phenomenon of Covid-19 Infected Cases in Indonesia with A Time Interval (0, 24) Hours from June to September 2020

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Abstract

Since the emergence of the 2019 novel coronavirus (2019-nCoV) in Wuhan City, China, in 2019, many countries in the world have been infected with the virus with very serious cases. This has caused a slump, especially for Indonesia, with a lower-middle-class society of 115 million people or 45% of the total population. The virus's rapid spread through droplets resulted in the Indonesian government implementing a lockdown called Large-scale Social Restrictions within months. This system is an effort to break the chain of virus spread. A pure birth process can be called a stochastic process with a continuous parameter space and a discrete state space. This research will focus on the application of the Pure Birth Process, which is expected to provide scientific studies on Covid-19 in Indonesia, become a new reference for the development of mathematics, and provide information on research results so that they can be used in policymaking to overcome and overcome the spread of Covid-19. At the end of this study, it was known that the spread of Covid-19 in Indonesia was very drastic, which had a rate of increase in cases as big as a person/hour or person/day, which is a very significant increase. Based on calculations using Pure Birth, it is hoped that it can be used in policymaking for the government and the community to overcome and overcome the spread of Covid-19.

Keywords: Pure birth application, stochastic process, Covid-19, infected cases

1. Introduction

Since the emergence of the 2019 novel coronavirus (2019-nCoV) in Wuhan City, China, in 2019 (Lu, et al., 2020), many countries in the world have been infected with the virus with very serious cases. On February 11, 2020, the World Health Organization (WHO) announced the new name of the virus to be 2019-nCoV: Coronavirus Disease (Covid-19). The International Committee on Taxonomy also decided that Covid-19 is a Severe Acute Respiratory Syndrome (SARS) with a very high spread speed. Until now, Covid-19 cases globally have reached 36,082,331 cases with a total death toll of 1,005,286 since November 17, 2019. This has caused a slump, especially for Indonesia, a country with a lower-middle-class society of 115 million people or 45% total population (Sugianto D, 2020).

The virus's rapid spread through droplets resulted in the Indonesian government implementing a lockdown system called Large-Scale Social Restrictions (PSBB) within months (Azanella, 2020). This system is an effort to break the chain of virus spread (Perdana et al, 2020).

In making policies to overcome and overcome the spread of Covid-19. A pure birth process can be called a stochastic process with a continuous parameter space and a discrete state space (Pinsky and Karlin, 2011). This research will focus on applying the Pure Birth Process, which is expected to provide scientific studies on Covid-19 in Indonesia, become a new reference for the development of mathematics, and provide information on research results to be used.

2. Materials and Methods

2.1. Materials

2.1.1. Research Secondary Data

Covid-19 for three months in Indonesia, starting from June 18, 2020, to September 18, 2020, resulting in a total of 93 data sets as presented in Table 1. This study takes secondary data, namely data obtained from the Worldometers web which was accessed on September 22, 2020. This data is data on the number of positive confirmed cases.

No	Month	Date	Number of Cases per Day	Cumulative Number of Cases
1	June	18	1331	1331
2		19	1041	2372
3		20	1226	3598
4		21	862	4460
5		22	954	5414
6		23	1051	6465
7		24	1113	7578
8		25	1178	8756
9		26	1240	9996
10		27	1385	11381
11		28	1198	12579
12		29	1082	13661
13		30	1293	14954
14	July	1	1385	16339
15		2	1624	17963
16		3	1301	19264
17		4	1447	20711
18		5	1607	22318
19		6	1209	23527
20		7	1268	24795
21		8	1853	26648
22		9	2657	29305
23		10	1611	30916
24		11	1671	32587
25		12	1681	34268
26		13	1282	35550
27		14	1591	37141
28		15	1522	38663
29		16	1574	40237
30		17	1462	41699
31		18	1752	43451
32		19	1639	45090
33		20	1693	46783
34		21	1655	48438
35		22	1882	50320
36		23	1906	52226
37		24	1761	53987
38		25	1868	55855
39		26	1492	57347
40		27	1525	58872
41		28	1748	60620
42		29	2381	63001
43		30	1904	64905
44		31	2040	66945
45	August	1	1560	68505
46		2	1519	70024
47		3	1679	71703
48		4	1922	73625
49		5	1815	75440
50		6	1882	77322
51		7	2473	79795
52		8	2277	82072
53		9	1893	83965
54		10	1687	85652
55		11	1693	87345
56		12	1942	89287
57		13	2098	91385
58		14	2307	93692
59		15	2345	96037
60		16	2081	98118
61		17	1821	99939
62		18	1673	101612
63		19	1902	103514
64		20	2266	105780
65		21	2197	107977
66		22	2090	110067
67		23	2037	112104
68		24	1877	113981
69		25	2447	116428
70		26	2306	118734
71		27	2719	121453
72		28	3003	124456
73		29	3308	127764
74		30	2858	130622
75		31	2743	133365
76	September	1	2775	136140
77		2	3075	139215
78		3	3622	142837
79		4	3269	146106
80		5	3128	149234
81		6	3444	152678
82		7	2880	155558
83		8	3046	158604
84		9	3307	161911
85		10	3861	165772
86		11	3737	169509
87		12	3806	173315
88		13	3636	176951
89		14	3141	180092
90		15	3507	183599
91		16	3963	187562
92		17	3635	191197
93		18	3811	195008

Table 1: Research Secondary Data

The number of daily positive confirmed cases of Covid-19 in the last three months has tended to rise, although there have been some declines on certain dates. The visualization of the increase is presented in Figure 3.1. Based on the graph above, it can be seen that the most cases occurred on September 16, 2020, reaching 3,963 cases, while the lowest cases were 862 on June 21, 2020. The plot of data based on the cumulative number of cases is presented in Figure 1.

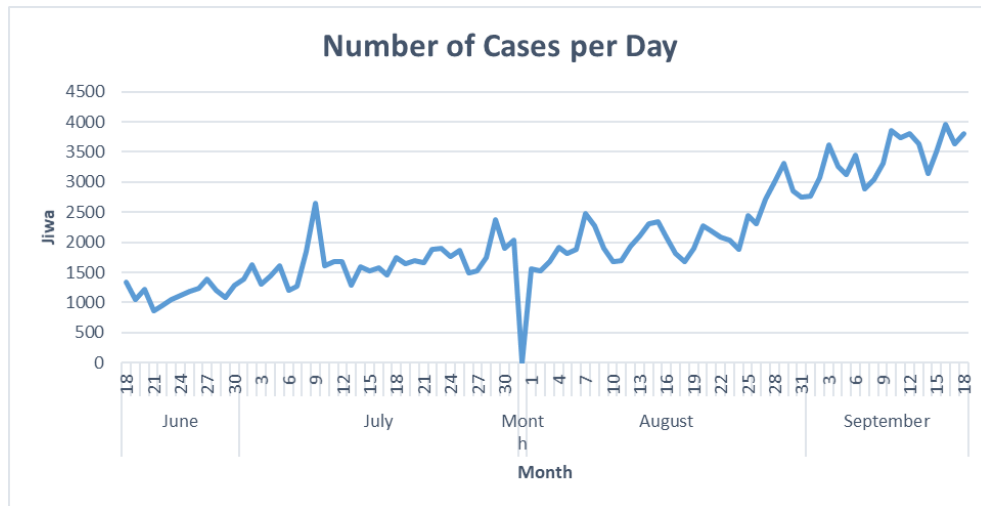


Figure 1: Number of Cases per Day

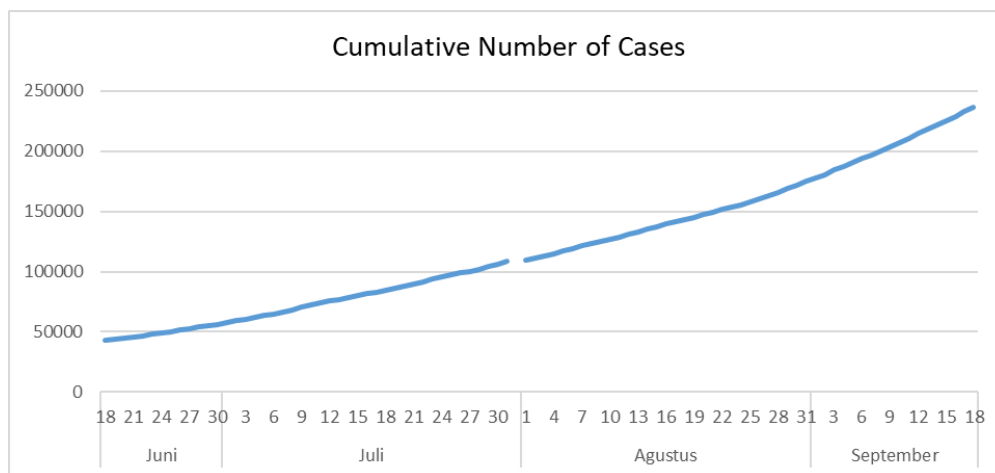


Figure 2: Cumulative Number of Cases

Based on the two Covid-19 data visualizations in Figures 3.1 and 3.2, it can be concluded that the secondary data obtained is stationary and up. In this study, the time interval (0.24) hours will be taken, a continuous parameter space with a discrete state-space represented in daily cumulative Covid-9 data collection. Therefore, these data can be applied to pure natal matter because it has a continuous parameter space and a discrete state space.

2.2. Methods

3.2.1. Stochastic Process

A Stochastic Process is defined as a family (sequence) of random variables given an index (Herberg, 1963). The stochastic process is defined by $X(t)$ where are the elements of the set corresponding to time,

$$X = \{X(t), t \in T\}. \quad (1)$$

A stochastic process is a sequence of events that satisfies the laws of probability (Karlin and Taylor, 1975). The stochastic process is widely used to model the evolution of a system that contains uncertainty or runs in an unpredictable environment (Orsingher and Polito, 2010). The deterministic model is no longer suitable for analyzing the system. The stochastic process appears in several events, such as:

- 1) Specific type of HP battery lifeline
- 2) The number of students who pass Calculus each year.
- 3) The number of incoming phone calls to one cell phone number.
- 4) Rows of water volume in a dam each day.

In this sequence, the uncertainty aspect appears dominant so that the system's evolution is better modeled as a stochastic process.

3.2.2. Classification and Examples of Stochastic Processes

From the Stochastic Process, it is known that the set T is called the parameter space (PS), while the set of all possible values of $X(t)$ is called the state space (SS) of the stochastic process X (Ding and Zhuang, 2014). Parameter space is divided into two, namely discrete and continuous parameter space (Costa and Alpuim, 2018). Discrete parameter space means a data set that is cumulative concerning time, for example, data obtained every day for one month. Meanwhile, continuous parameter space means data collection obtained in the form of time intervals, for example, data obtained in time intervals (0.24) hours. Like the parameter space, the state space is divided into discrete and continuous. An example of discrete state space is the number of visitors in Store A, while a continuous state space is an example of the room temperature in Store A.

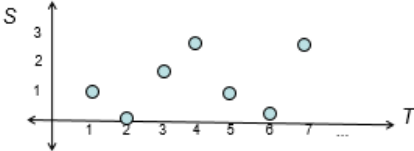
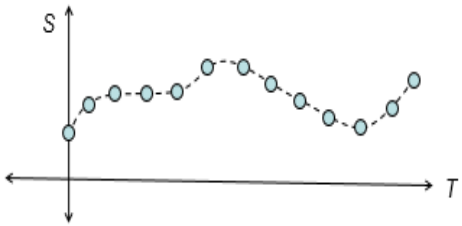
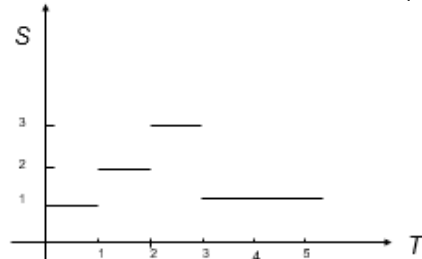
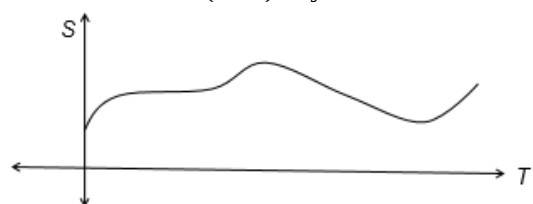
SS PS	Discrete	Continue
Continue	○ The number of cellphones sold in a shopper day ○ The number of bacteria in the nth generation ○ The number of Unpad Mathematics Department students who come to campus every day 	○ The volume of rainfall per day in one month ○ Vehicle speed on Padaleunyi toll road every Sunday 
Discrete	○ Number of visitors to shop "P" in interval (0.30) days ○ The number of vehicles entering the Unpad gate at an interval of time (0.24) hours 	○ Air temperature in Bandung in intervals (0.24) hours ○ The volume of formula milk taken by baby A at intervals of (0.30) days 

Table 2: Classification and Examples of Stochastic Processes

3.2.3. Poisson Process as an example of a Pure Birth Process

According to Karlin and Taylor (1975), the stochastic process with discrete state space and continuous parameter space is a mathematical model often encountered in everyday life. Forms like this include the Poisson process, an example of a pure birth process. The Poisson process is an example of a Point Process (taking points from an event), a stochastic process with realization in the form of a counting process. During the Poisson process, the process of calculating the number of events in a certain time interval is carried out.

Stochastic process $\{N(t), t \geq 0\}$ is said to be a counting process if $N(t)$ represents the number of events that occur during a time. For example, if the $N(t)$ is the number of babies born during time t , then $\{N(t), t \geq 0\}$ is the counting process and if $N(t)$ is the number of people come to the supermarket in the time interval (0,24) hour, then $\{N(t), t \geq 0\}$ is a counting process.

A counting process $\{N(t), t \geq 0\}$ is said to be a Poisson process with a parameter rate $\lambda > 0$ if it satisfies the following conditions:

- 1) $N(0) = 0$
- 2) Stationary independent increments
- 3) $P(N(h) = 1) = \lambda + o(h)$
- 4) $P(N(h) \geq 2) = o(h)$

Since the Poisson process has a stationary independent increment, then:

$$P(N(s+t) - N(s) = k) = P(N(t) = k | N(0) = 0) = P_k(t), \text{ for any } s \geq 0 \text{ and } t \geq 0. \quad (2)$$

where $P_k(t)$ is the probability that an event occurs in the time interval t , with the formula:

$$P_k(t) = \frac{(\lambda t)^k (e)^{-\lambda t}}{k!}, k = 0, 1, 2, 3, \dots \quad (3)$$

The inter-arrival time X_n , $n = 1, 2, 3, \dots$ of a Poisson process is independent and has an exponential distribution with the parameter λ .

$$P(X_k \leq t) = 1 - P(X_k > t) = 1 - P(N(t) = 0) = 1 - e^{-\lambda t}, \quad (4)$$

$$P(X_k \leq t) = 1 - e^{-\lambda t}. \quad (5)$$

The realization of the pure birth process is presented in Figure 3.

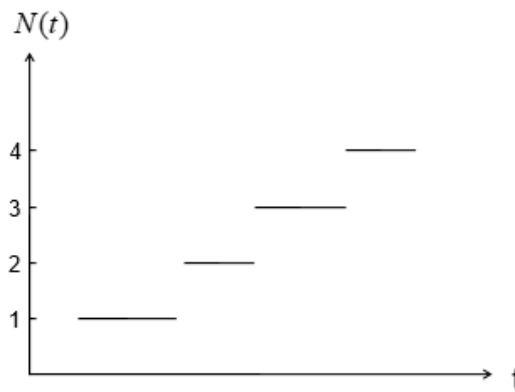


Figure 3: The realization of the pure birth process

Let $P_{ij}(t) = P(N(t) = j | N(0) = i)$, $i = 0, 1, 2, 3, \dots$ be the probability of a stationary transition from state i to state j . So if given the initial conditions $N(0) = 0$, then $P_k(t) = P(N(t) = k | N(0) = 0)$, $k = 0, 1, 2, 3, \dots$ states the probability that an event will occur in the interval $(0, t]$. It is also known that $P_0(t) = e^{-\lambda_0 t}$. There are also supporting theorems for the pure birth process, including:

1) Theorem 1

For pure birth processes $\{N(t), t \geq 0\}$ with parameters λ_k , $k = 0, 1, 2, \dots$, then the time between arrivals (time between births) X_{k+1} , $k = 0, 1, 2, 3, \dots$ is independent and has an exponential distribution with parameters λ_k and mean $\frac{1}{\lambda_k}$.

2) Theorem 2

For a pure birth process $\{N(t), t \geq 0\}$ with parameters λ_k , $k = 0, 1, 2, \dots$, then it holds $\sum_{k=0}^{\infty} P_k(t) = 1, \forall t \geq 0$ if and only if $\sum_{k=0}^{\infty} \frac{1}{\lambda_k} = \infty$.

3. Results and Discussion

4.1. Data Processing

This chapter discusses the application of the Pure Birth Process to Covid-19 data in Indonesia from June 18, 2020, to September 18, 2020, with an interval of (0.24) hours. Secondary data was taken from web Worldometers (<https://www.worldometers.info/coronavirus/>) accessed on September 22, 2020.

Based on the data obtained, using data on the number of cases per day, the pure birth process in question is the process of increasing the number of confirmed cases of Covid-19 for a total of 93 study days and a total of 195,0008 cases. First of all, we will determine λ first. Noted that $E(N(t)) = \lambda t$ is the expected amount of increase, then:

$$\lambda = \frac{E(N(t))}{t}. \quad (6)$$

Since this research is an application of a Pure Birth Process, a Stochastic Process with a continuous parameter space, the time interval (0.24) hours is determined as a continuous parameter space of a total of 93 days. So the value of t is the multiplication of the number of hours/the upper limit of the time interval to the number of research days as below:

$$t = \text{time interval upper limit} \times \text{number of day} = 24 \times 93 = 2232, \quad (7)$$

with $E(N(t))$ the value of the average calculation of the data is as follows,

$$E(N(t))_1 = \frac{\text{total data}}{\text{jumlah data (hari)}} = \frac{195008}{93} = 2097.72043 \approx \frac{2098 \text{ people}}{\text{day}}, \quad (8)$$

or in other words,

$$E(N(t))_2 = \frac{2098}{24} = 87.4166666667 \approx \frac{88 \text{ paper}}{\text{hour}}. \quad (9)$$

Substitute the value of $E(N(t))$ and t to the first equation,

$$\lambda_1 = \frac{E(N(t))_1}{t} = \frac{2098}{2232} = 0.9399 \approx 1 \text{ person/hour or } \lambda_2 = 1 \times 24 = 24 \text{ people/day}. \quad (10)$$

So, the increase in cases per unit of time is one person/hour or 24 people/day.

4.2. Application of Data to Problems

Noted that:

$$P_{x(t)} = \frac{(\lambda t)^x e^{-\lambda t}}{k!}, k = 0, 1, 2, \dots \quad (11)$$

In the Covid-19 case data, from July 18 to September 18, 2020, there were as many as 195,088 confirmed positive cases of Covid-19 in Indonesia. The increase in cases followed the Poisson process with a birth rate/unit of time of 1 person/hour. Will determine:

- 1) Chances of no additional cases in one day.

Answer:

$$P_k(t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, k = 0, 1, 2, 3, \dots \quad (12)$$

Noted that $\lambda = 24$ people/day.

Suppose $N(t)$ is the number of people the increase in cases in time t (day), then $t = 1$,

$$P_0(1) = P(N(1) = 0) = \frac{(24 \cdot 1)^0 \cdot e^{-24 \cdot 1}}{0!} = e^{-24} = 0,00000000004 \approx 0. \quad (13)$$

So, the probability of no additional cases in 1 day is 0.

Interpretation: It is impossible not to increase the number of Covid-19 cases in one day.

- 2) If in 3 days the increase in cases is 45 people, determine the probability of increasing cases in the last one day if there have been 40 people added in the first two days.

Answer:

Noted that $\lambda = 1$ person/hour, then

$$P(N(3) - N(2) = 45 - 40) = P(N(1) = 5). \quad (14)$$

Then,

$$P_5(1) = \frac{(1 \cdot 1)^5 \cdot e^{-1 \cdot 1}}{5!} = \frac{(1)^5 \cdot e^{-1}}{5!} = 0,0228738689. \quad (15)$$

So, the probability is 0.0228738689.

Interpretation: The increase in Covid-19 cases in the last 1 hour is approaching 2.28%.

- 3) The probability that the time interval between the addition of the 4th case and the 5th case is not more than a day.

Answer:

$$P_k(t) = \frac{(\lambda t)^k (e)^{-\lambda t}}{k!}, k = 0, 1, 2, 3, \dots \quad (16)$$

Noted that $\lambda = 1$ person/hour and let the time between cases increase is $X(t)$, then:

$$P(X_k \leq t) = 1 - P(X_k > t) = 1 - P(N(t) = 0) = 1 - e^{-\lambda t}, \quad (17)$$

$$P(X_5 \leq 0,5) = 1 - P(X_5 > 0,5) = 1 - P(N(0,5) = 0) = 1 - P_0(0,5) = 0,39346934029. \quad (18)$$

So, the chance of adding cases 4 and 5 cases of Covid-19 in no more than an hour is 39.34%.

4. Conclusions

In this study, it can be concluded that the spread of Covid-19 in Indonesia is very drastic, which has a rate of increase in cases of 1 person/hour or 24 people/day, which is a very significant increase. Based on calculations using Pure Birth Process, it is hoped that it can be used in policymaking for the government and the community to overcome and overcome the spread of Covid-19.

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