



## Calculation Option Price on Shares PT. Nippon Indosari Corpindo Tbk. with Fuzzy Binomial Tree Model

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### Abstract

The Cox-Ross-Rubinstein binomial tree model is widely used in stock and derivative securities calculations, such as options calculations. The binomial CRR model assumes that the parameter increases in option prices and decreases in option prices so this model produces stock price movements up and down stock price movement. However, stock price movements show price fluctuations and cause the volatility of the value to be unsuitable. In this study, modeling stock and option price movements using a fuzzy binomial tree model. The data used was data on the movement of the stock price of Nippon Indosari Corpindo Ltd Plc from February 2021 to January 2022. The results showed that for February 2022, with a risk size of 90%, the selling price options with the greatest volatility of 51.6484081, medium volatility of 33.33154354, and the smallest volatility of 28.17155892.

**Keywords:** CRR binomial model, fuzzy binomial model, fuzzy put option, fuzzy volatility

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### 1. Introduction

According to Singh (2012) investment is a commitment to a certain amount of funds or other resources made now with the hope of obtaining benefits in the future. The parties who carry out investment activities are usually called investors. Investors can generally be classified into two, namely individual investors and institutional investors. Institutional investors consist of insurance companies, savings institutions, pension funds, and investment companies. Institutions like this usually collect money from their customers for investment or can also buy shares or bonds. Shares are the rights that people have to a company thanks to the transfer of a share of capital so that they are considered to share in ownership and supervision (Cai et al., 2013). Stock options as derivatives are a financial instrument that is expected to safeguard the risk of the shares concerned. The most important thing in options trading is determining the optimal selling price for options. However, in reality, share price fluctuations that occur on the stock exchange cause investors to experience difficulties in determining the optimal option selling price.

According to Cox et al. (1979) a simple approach to calculating option prices, namely a discrete time option pricing formula. To get option prices that are close to the Black Scholes continuous model, it takes quite a long time because, with more time partitions, the process of calculating option prices will also increase. One method used to speed up the convergence of Binomial CRR method option prices to Black Scholes method option prices is to smooth the curve on the CRR binomial tree. In research (Mustafa et al., 2022), the volatility parameter for stock price movements uses a membership function that has a membership degree between 0 and 1 with sensitivity analysis only when  $t = 1$ . Therefore, the author is interested in estimating the put option price parameters using sensitivity analysis at time  $t = 2$  to see more convergent results. This estimate will be made for the share price of PT. Nippon Indosari Corpindo Tbk.

### 2. Materials and Methods

This research article was completed using literature studies and case studies. The data used in this research is the share movement data of PT. Nippon Indosari Corpindo Tbk. for the period February 2021 to January 2022. The put option price calculation was completed for one period using a fuzzy binomial model with the help of Microsoft Excel 2013 software.

### 3. Results and Discussion

#### 3.1. Parameter Estimation of Fuzzy Binomial Model

In the binomial CRR model, it is assumed that at one time, the stock price will increase with a parameter of  $u$  with probability  $p$  and will decrease of  $d$  with probability  $(1 - p)$ . According to Milanesi et al. (2015) to estimate rising and falling parameters with a triangular curve representation in this case produces

$$u = e^{\sigma\sqrt{\Delta T}}, \quad d = e^{-\sigma\sqrt{\Delta T}}, \quad p = \frac{e^{r\Delta T} - d}{u - d}$$

#### 3.2. Parameter Estimation during Upward Movement

According to Yu et al. (2011)  $uu$ ,  $um$ , and  $ud$  are three possible upward movements at the largest, medium, and smallest volatility. The fuzzy binomial model only has one volatility which is used to calculate the put option during an upward movement, so the parameter for increasing the option price is only  $u$  (firm value).

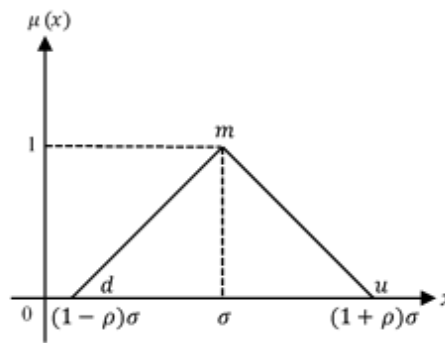


Figure 1: Fuzzy triangle curve during an upward movement

By combining the parameter  $u$  and the triangular curve, we obtain the put option parameters in the fuzzy binomial model is

$$uu = e^{(1+\rho)\sigma\sqrt{\Delta T}}, \quad um = e^{\sigma\sqrt{\Delta T}}, \quad ud = e^{(1-\rho)\sigma\sqrt{\Delta T}} \quad (1)$$

#### 3.3. Parameter Estimation during Downward Movement

To find out the parameters of the fuzzy binomial model during a downward movement, when the stock price at  $t = 1$  is known. This can be illustrated by a fuzzy binomial tree through a symmetrical triangular curve representation as follows.

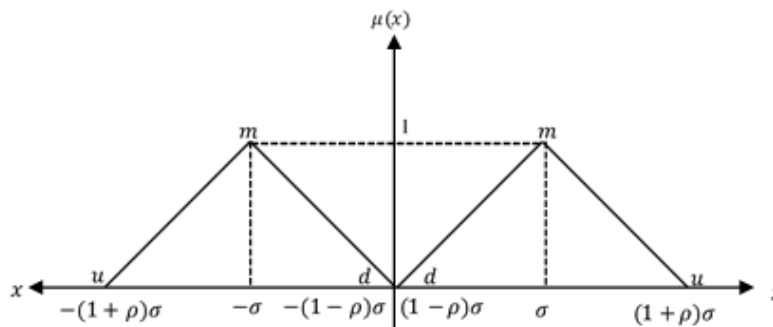


Figure 2: Reflection of the fuzzy triangle curve during upward movement

The representation of the reflection of a fuzzy triangular curve during an upward movement, based on the binomial CRR model when experiencing a downward price movement, has only one parameter is  $d$  (firm value). Then the selling option parameters are obtained in the fuzzy binomial model is

$$du = e^{-(1-\rho)\sigma\sqrt{\Delta T}}, \quad dm = e^{-\sigma\sqrt{\Delta T}}, \quad dd = e^{-(1+\rho)\sigma\sqrt{\Delta T}} \quad (2)$$

### 3.4. Volatility of Stock Price

According to Mawardi (2018), volatility is a change in share prices due to increased demand for shares or many investors selling shares. Following are the steps to find volatility:

- Calculate the return at time  $t$  using the formula  $R_t = \ln(\frac{S_t}{S_{t-1}})$ , with  $S_t$  is stock price at time  $t$  for  $t = 1, 2, \dots, 12$ .
- Calculate the average return using the formula  $\bar{R} = \frac{1}{n} \sum_{t=1}^n R_t$ , obtained  $\bar{R} = 0.0128197$ .
- Calculate the variance using the formula  $s^2 = \frac{1}{n-1} \sum_{t=1}^n (R_t - \bar{R})^2$ , obtained  $s^2 = 0.001807792$ .
- Calculate annual volatility using the formula  $\sigma = \sqrt{12 \times s^2}$ , obtained  $\sigma = 0.147287134$ . So the volatility for share price movements at PT. Nippon Indosari Tbk. from February 2021 to January 2022 is 0.147287134.

### 3.5. Determine Fuzzy Put Option Price at time $t = 1$

Based on data on share movements of PT. Nippon Indosari Tbk., obtained  $S = 1355$ ,  $K = 1345$ ,  $\Delta T = 0.08333$ ,  $r = 3.50\%$ ,  $a = e^{r\Delta T} = 1.002920924$  from equation (1) and (2):

$uu = 1.084137189$ ,  $um = 1.043434977$ ,  $ud = 1.004260865$

$du = 0.995757213$ ,  $dm = 0.958373087$ ,  $dd = 0.922392489$

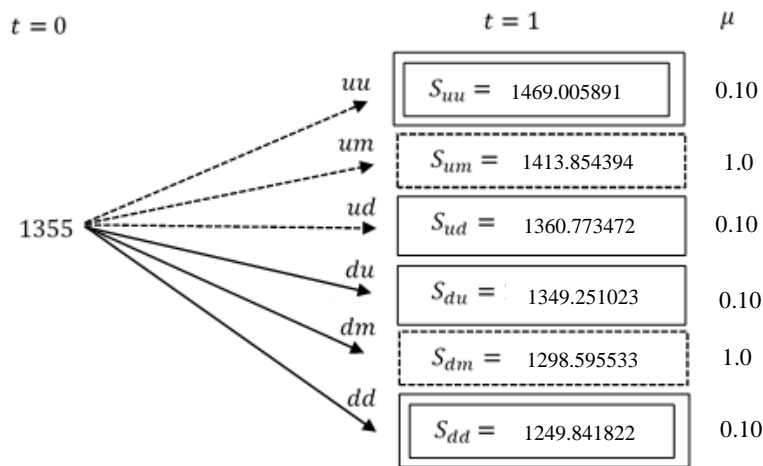


Figure 3: Fuzzy binomial tree for stock prices at time  $t = 1$

In looking for all possibilities in stock prices, we can define the price determination of the put option at time  $t = n$  (last period) which can be expressed as

$$P_{uu \cdot h} = \max\{0, K - uu \cdot S_h\} \quad (3)$$

$$P_{um \cdot h} = \max\{0, K - um \cdot S_h\} \quad (4)$$

$$P_{ud \cdot h} = \max\{0, K - ud \cdot S_h\} \quad (5)$$

$$P_{du \cdot h} = \max\{0, K - du \cdot S_h\} \quad (6)$$

$$P_{dm \cdot h} = \max\{0, K - dm \cdot S_h\} \quad (7)$$

$$P_{dd \cdot h} = \max\{0, K - dd \cdot S_h\} \quad (8)$$

Then, using the definition of pricing the put option to be executed in February 2022 at time  $t = 1$  is as follows.

$$P_{uu \cdot h} = 0$$

$$P_{um \cdot h} = 0$$

$$P_{ud \cdot h} = 0$$

$$P_{du \cdot h} = 0$$

$$P_{dm \cdot h} = 46.40446732$$

$$P_{dd \cdot h} = 95.15817793$$

Each fuzzy option value at  $t = n - 1$  is formed into  $P_h^r$  formed from two option values with the largest volatility,  $P_h^c$  formed from two option values with medium volatility, and  $P_h^l$  formed from two option values with the smallest volatility. In the binomial CRR model to obtain the fuzzy option value using the model in equations (3) to (8) and combined with the put option formula, we obtain

$$P_h^r = e^{-r\Delta T} \left[ \frac{(a-dd) \cdot C_{uu \cdot h} + (uu-a) C_{dd \cdot h}}{(uu-dd) \cdot a} \right] \quad (9)$$

$$P_h^c = e^{-r\Delta T} \left[ \frac{(a-dm) \cdot C_{um \cdot h} + (um-a) C_{dm \cdot h}}{(um-dm) \cdot a} \right] \quad (10)$$

$$P_h^l = e^{-r\Delta T} \left[ \frac{(a-du) \cdot C_{ud \cdot h} + (ud-a) C_{du \cdot h}}{(ud-du) \cdot a} \right] \quad (11)$$

By using equations (9) to (11), the fuzzy put option price is obtained with value  $P^r$  is 47.64226254,  $P^c$  is 22.03757283, and  $P^l$  is 0.

### 3.6. Determine Fuzzy Put Option Price at time $t = 2$

Next, we will look for the price of the fuzzy put option at time  $t = 2$  and  $\rho = 90\%$ , obtained  $S = 1355$ ,  $K = 1345$ ,  $\Delta T = 0.041666667$ ,  $r = 3.50\%$ ,  $a = e^{r\Delta T} = 1.001459397$  from equation (1) and (2):

$uu = 1.058786282$ ,  $um = 1.030521372$ ,  $ud = 1.00301101$

$du = 0.996998029$ ,  $dm = 0.970382592$ ,  $dd = 0.94447767$

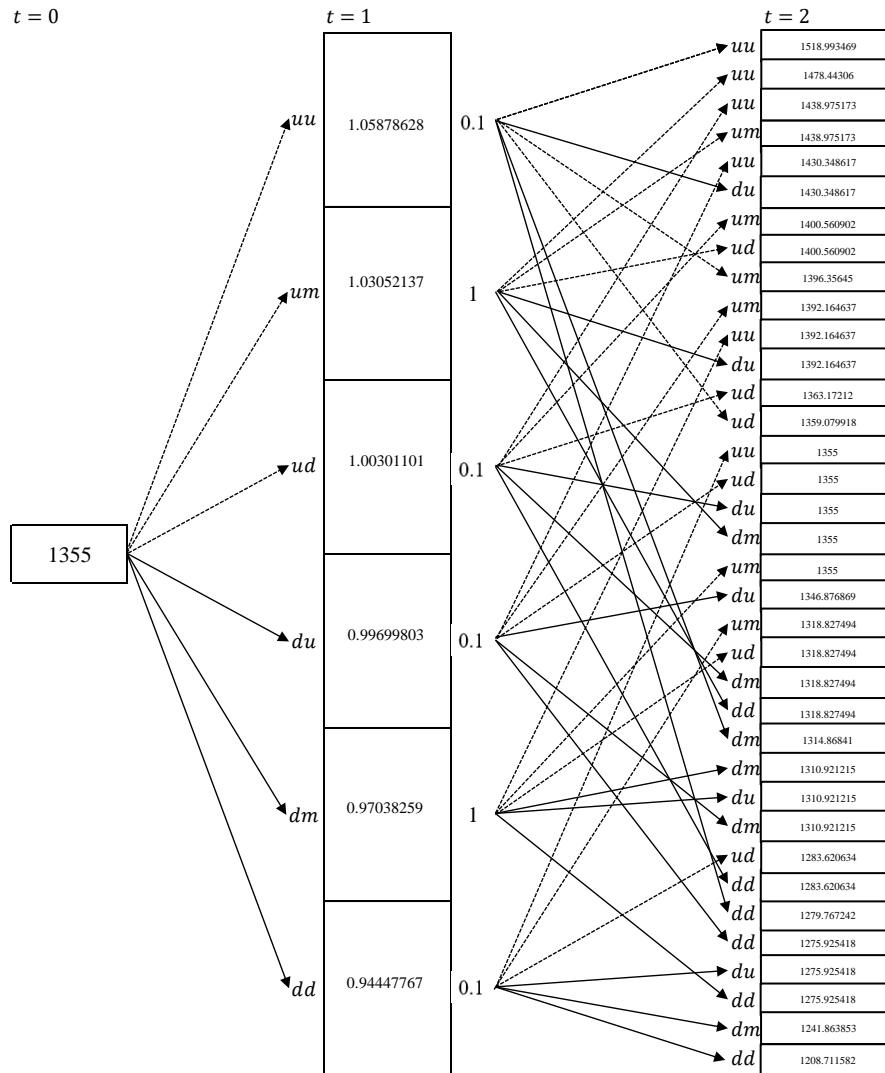


Figure 4: Fuzzy binomial tree for stock prices at time  $t = 2$

Then, using the definition of pricing the put option to be executed in February 2022 at time  $t = 2$  is as follows.

$$\begin{aligned}
 P_{uu \cdot h^1} &= 0 \\
 P_{um \cdot h^1} &= 0 \\
 P_{ud \cdot h^1} &= 0 \\
 P_{du \cdot h^1} &= 0 \\
 P_{dm \cdot h^1} &= 0 \\
 P_{dd \cdot h^1} &= 0 \\
 P_{uu \cdot h^2} &= 0 \\
 P_{um \cdot h^2} &= 0 \\
 P_{ud \cdot h^2} &= 0 \\
 P_{du \cdot h^2} &= 0
 \end{aligned}$$

$$\begin{aligned}
P_{dm \cdot h^2} &= 0 \\
P_{dd \cdot h^2} &= 26.17250571 \\
P_{uu \cdot h^3} &= 0 \\
P_{um \cdot h^3} &= 0 \\
P_{ud \cdot h^3} &= 0 \\
P_{du \cdot h^3} &= 0 \\
P_{dm \cdot h^3} &= 26.17250571 \\
P_{dd \cdot h^3} &= 61.37936554 \\
P_{uu \cdot h^4} &= 0 \\
P_{um \cdot h^4} &= 0 \\
P_{ud \cdot h^4} &= 0 \\
P_{du \cdot h^4} &= 0 \\
P_{dm \cdot h^4} &= 34.7878468 \\
P_{dd \cdot h^4} &= 69.07458184 \\
P_{uu \cdot h^5} &= 0 \\
P_{um \cdot h^5} &= 0 \\
P_{ud \cdot h^5} &= 26.17250571 \\
P_{du \cdot h^5} &= 34.07878468 \\
P_{dm \cdot h^5} &= 69.07458184 \\
P_{dd \cdot h^5} &= 103.136146
\end{aligned}$$

Thus, the fuzzy put option price is obtained using equations (9) to (11) is

$$\begin{aligned}
P_{h^1}^r &= 0 \\
P_{h^1}^c &= 0 \\
P_{h^1}^l &= 0 \\
P_{h^2}^r &= 13.10663924 \\
P_{h^2}^c &= 0 \\
P_{h^2}^l &= 0 \\
P_{h^3}^r &= 30.73749261 \\
P_{h^3}^c &= 12.62939261 \\
P_{h^3}^l &= 0 \\
P_{h^4}^r &= 34.59109474 \\
P_{h^4}^c &= 16.44452221 \\
P_{h^4}^l &= 0 \\
P_{h^5}^r &= 51.64840819 \\
P_{h^5}^c &= 33.33154354 \\
P_{h^5}^l &= 28.17155892 \\
P_{h^6}^r &= 68.25036684 \\
P_{h^6}^c &= 50.57992868 \\
P_{h^6}^l &= 63.27272878
\end{aligned}$$

### 3.7. Sensitivity Analysis of Put Option Price Fuzzy

The sensitivity analysis calculation for this study was carried out at  $t = 2$  using a risk measure of 90%, medium risk of 50%, and very low risk of 10%. Based on fuzzy theory, the results obtained are as shown in Table 1.

**Table 1:** Put option price fuzzy with different risk measures

		Sensitivity Analysis at $t = 1$		Sensitivity Analysis at $t = 2$			
		Index		Index			
		1	2	3	4	5	
$\rho$	10%	$P^l$	19.2109972	0	29.40689718	29.40689718	31.25306001
		$P^c$	22.03757283	0	0	31.47769326	33.33154354
		$P^r$	24.87080859	0	0	33.53558476	35.39518423
	50%	$P^r$	8.054841514	0	13.69645971	13.69645971	28.17155892
		$P^c$	22.03757283	0	4.929966519	24.00630421	33.33154354
		$P^l$	36.24216005	5.050845305	14.89632967	34.14880599	43.56014857
	90%	$P^r$	0	0	0	0	28.17155892
		$P^c$	22.03757283	0	12.62939261	16.44452221	33.33154354
		$P^l$	47.64226254	13.10663924	30.73749261	34.59109474	51.64840819

In Table 1 it can be seen that  $\rho$  with the largest percentage has the largest interval, so the smaller the value of  $\rho$  the interval between the  $P^r$  and  $P^l$  values is also smaller. Sensitivity analysis at  $t = 1$  has only one option value, while at  $t = 2$  there are many option values and the values are increasingly convergent and if the sensitivity analysis ( $\rho$ ) is greater, the risk accepted is also greater. From the analysis that has been carried out, the decisions that investors can choose can be seen in Figure 5.

		Dare to take risks	Neutral to risk	Very risk averse
$r$ → $c$ → $l$ →	(a)	Put (strong) 51.64840819	Put 43.56014857	Put 35.39518423
	(b)	Put 33.33154354	Put 33.33154354	No transaction
	(c)	Call 28.17155892	Put 28.17155892	No transaction
	(d)	Call (strong)	Put	Call 31.25306001

**Figure 5:** Sensitivity analysis decisions

$r$  : the price of the put option with the greatest volatility, which indicates investors can carry out strategy (a) or (b)  
 $c$  : the price of a put option with moderate volatility, which indicates investors can carry out strategy (b) or (c)  
 $l$  : the price of the put option with the smallest volatility, which shows investors can carry out strategy (c) or (d)

In Figure 5 it is explained that investors can choose the option price with the greatest volatility  $\rho = 90\%$  or investors who dare to take the risk in exercising their option rights without hesitation with a put option price value of 51.64840819, it is highly recommended to sell, an option price of 33.33154354 is recommended to sell. sell or buy, and the option price is 28.17155892, it is highly recommended to buy. Then, neutral investors can exercise their option rights when volatility is greatest and moderate. Investors who avoid risk can exercise their option rights by choosing the option price with the greatest volatility and avoiding making transactions during times of medium volatility.

#### 4. Conclusion

From data on share price movements of PT. Nippon Indosari Corpindo Tbk. for the period February 2021 to January 2022, using the fuzzy binomial method, the price of the put option was obtained with a risk size of 90% is each share to be purchased will experience increases and decreases, namely for the greatest volatility obtained  $P^r = 51.6484081$ , for medium volatility obtained  $P^c = 33.33154354$ , and for the smallest volatility obtained  $P^l = 28.17155892$ .

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