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Weibull Mortality Tables Based on Indonesian Mortality Table 2019

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Abstract

Life tables are used to calculate the probability that a member of a population for each age group in a certain country would survive until the next year and die before the next year. A life table could be constructed based on another life table using a mortality law. This study will construct a Weibull Life Table based on the 2019 Indonesian Life Table using Weibull's mortality law, with its parameters estimated using the Ordinary Least Squares method. The results of this study show that the oldest age in the male Weibull Life Table is 115 years, which is 4 years older than the oldest age in the male 2019 Indonesian Life Table. The oldest age in the female Weibull Life Table is 119 years, which is 8 years older than the older age in the female 2019 Indonesian Life Table.

Keywords: Indonesian life table 2019, Weibull's mortality law, Ordinary Least

1. Introduction

Mortality tables present population mortality data for each age group in a country. According to Mehr and Gustavson (1987:620), mortality tables also calculate the chances that a person will remain alive within a certain period. The mortality table used in this research is the 2019 Indonesian Mortality Table, and the mortality table was obtained using the Weibull mortality law to calculate mortality data.

According to Sembiring (in Anderson, 2004), a person's chances of living and dying can be calculated using the law of mortality. Several mortality laws can be used to calculate these opportunities, namely De Moivre, Gompertz, Makeham, and Weibull. In this research, the Weibull mortality law will be used, which is a parametric distribution used in survival models and is related to survival (Kleinbaum & Klein, 2012: 290).

The data presented in the mortality table influences the actuarial values for calculating the premium that must be paid in life insurance. Previous research has calculated actuarial values based on mortality tables created using mortality laws. Chen's (2018) research regarding the calculations of the annual premium value of endowment life insurance using the Weibull mortality law using stochastic interest rates found that insurance premiums were lower than using the 2011 Indonesian Mortality Table in general. Nelso's (2019) research on determining combined endowment insurance premium reserves using the Weibull mortality law in the New Jersey method found that there was an increase in the value of premium reserves at the end of the first year until the following year. Research by Anderson. (2004) regarding determining endowment insurance premiums using the Illinois method based on the Weibull mortality law found that the policy value is greater than the policy value using the Illinois method was not based on the Weibull mortality law the first year to the 20th year.

Kleinbaum and Klein (2012:304) write that the Weibull distribution density function is denoted as $f(x\colon \alpha, \lambda)$, with

$$f(x\colon \alpha, \lambda) = \lambda \alpha x^{\alpha-1} \exp(-\lambda x^{\alpha}), \tag{1}$$

where the random variable x is time, the parameter α is the shape parameter and the parameter λ is the scale parameter, and $x > 0$, $\alpha, \lambda > 0$. The cumulative probability function of the Weibull distribution from Equation (1) is

$$F(x\colon \alpha, \lambda) = 1 - \exp(-\lambda x^{\alpha}), \tag{2}$$

with x > 0, α, λ > 0. Thus, the Weibull distribution survival function from Equation (1) is

$$S(x:\alpha,\lambda) = \exp(-\lambda x^{\alpha}), \tag{3}$$

With $x > 0$, $\alpha, \lambda > 0$.

If the parameter $\lambda = \beta^{-\alpha}$ is a scale parameter, then the Weibull distribution density function from Equation (1) becomes

$$f(x:\alpha,\beta) = \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], \tag{4}$$

with x > 0, α, λ > 0. Then, the Weibull cumulative distribution function from Equation (4)

$$F(x:\alpha,\beta) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], \tag{5}$$

with x > 0, α, λ > 0. Thus, the Weibull distribution survival function from Equation (4) is

$$S(x;\alpha,\beta) = \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], \tag{6}$$

With $x > 0$, $\alpha, \lambda > 0$. Based on Equations (4) and (6), the death rate in the Weibull distribution is

$$\begin{aligned}\mu_x &= \frac{f(x)}{S(x)}, \\ &= \frac{\frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]}{\exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]}, \\ &= \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1},\end{aligned} \tag{7}$$

With $x > 0$, $\alpha, \lambda > 0$.

Based on Equation (7), the death rate for someone aged x years who will remain alive until $x + n$ years is

$$\mu_{x+n} = \frac{\alpha}{\beta^{\alpha}} (x+n)^{\alpha-1}. \tag{8}$$

Furthermore, the probability that someone aged x years will live until $x + n$ years is

$$\begin{aligned}{}_np_x &= \exp\left[-\int_0^n \mu_{x+s} ds\right]. \\ &= \exp\left[-\int_0^n \frac{\alpha}{\beta^{\alpha}} (x+s)^{\alpha-1} ds\right]. \\ &= \exp\left[-\frac{\alpha}{\beta^{\alpha}} \int_0^n (x+s)^{\alpha-1} ds\right].\end{aligned} \tag{9}$$

Suppose $u = (x+s)$ and $du = ds$, then Equation (9) can be reduced to

$${}_np_x = \exp\left[-\frac{\alpha}{\beta^{\alpha}} \left(\frac{1}{\alpha} u^{\alpha} |_0^n\right)\right]. \tag{10}$$

Then, by substituting $u = (x+s)$ into Equation (10), we obtain

$$\begin{aligned}{}_np_x &= \exp\left[-\frac{\alpha}{\beta^{\alpha}} \left(\frac{1}{\alpha} (x+s)^{\alpha} |_0^n\right)\right]. \\ &= \exp\left[-\frac{\alpha}{\beta^{\alpha}} \left(\frac{1}{\alpha} ((x+s)^{\alpha} - x^{\alpha})\right)\right]. \\ &= \exp\left[-\left(\frac{(x+n)^{\alpha} - x^{\alpha}}{\beta^{\alpha}}\right)\right].\end{aligned} \tag{11}$$

The probability that a person aged x years will die before the age of $x + n$ years is

$$\begin{aligned}{}_nq_x &= 1 - {}_np_x. \\ &= 1 - \exp\left[-\left(\frac{(x+n)^{\alpha} - x^{\alpha}}{\beta^{\alpha}}\right)\right].\end{aligned} \tag{12}$$

2. Materials and Methods

Weibull distribution parameter estimation was carried out using the scale parameter $\lambda = \beta^{-\alpha}$ and the parameter α (Kleinbaum and Klein, 2012:304). The survival function in the Weibull distribution has a linear relationship with age. This relationship is

$$\begin{aligned}s(x) &= \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]. \\ \ln s(x) &= \ln\left(\exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]\right). \\ \ln s(x) &= -\left(\frac{x}{\beta}\right)^{\alpha}. \\ -\ln s(x) &= \left(\frac{x}{\beta}\right)^{\alpha}.\end{aligned}$$

$$
\begin{aligned}
\ln[-\ln s(x)] &= \ln\left(\frac{x}{\beta}\right)^{\alpha}.\\
&= \ln\left(\frac{x^{\alpha}}{\beta^{\alpha}}\right).\\
&= \ln x^{\alpha} - \ln \beta^{\alpha}.\\
&= \alpha \ln x - \alpha \ln \beta. \qquad (13)
\end{aligned}
$$

In Equation (13), $\ln[-\ln s(x)]$ is a linear function of $\ln(x)$with α being the slope and $\ln(\lambda)$ being the intercept. If the slope is 1, then X follows an exponential distribution.

The Ordinary Least Squares method based on Equation (13) is used to obtain parameter values. Suppose there is a linear function as follows

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i \, ; \qquad i = 1, 2, \dots, n \qquad (14)$$

Suppose there are n data and two random variables X_1 and X_2. Assume that X_1 and X_2 do not have the distribution $\varepsilon_i \sim N(0, \sigma^2)$. β_0, $\beta_1$and β_2 are estimated, by denoting the coefficients by b_0, b_1 dan b_2.

$$
\begin{aligned}
\mathrm{J} &= \textstyle\sum_{i=1}^{n} \varepsilon_i^2.\\
&= \textstyle\sum_{i=1}^{n} (y_1 - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2})^2. \qquad (15)
\end{aligned}
$$

The minimum is obtained by finding the derivative J with respect to β_0, β_1 and $\beta_2$then equating each derivative to zero. Equation (15) if reduced to β_0 can be written as follows

$$
\begin{aligned}
\frac{\partial \mathrm{J}}{\partial \beta_0} &= -2\Sigma(y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2}).\\
&= 0. \qquad (16)
\end{aligned}
$$

Equation (15) if derived from $\beta_1$can be written as follows

$$
\begin{aligned}
\frac{\partial \mathrm{J}}{\partial \beta_1} &= -2\Sigma(y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i1}.\\
&= 0. \qquad (17)
\end{aligned}
$$

Equation (15) if derived from β_2 can be written as follows

$$
\begin{aligned}
\frac{\partial \mathrm{J}}{\partial \beta_2} &= -2\Sigma(y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i2}.\\
\frac{\partial \mathrm{J}}{\partial \beta_2} &= -2\Sigma(y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i2}.\\
&= 0. \qquad (18)
\end{aligned}
$$

In the following calculations β_0, β_1 and $\beta_2$are substituted with b_0, b_1 and b_2. Thus, Equation (16) can be simplified as

$$nb_0 + b_1 \Sigma x_{i1} + b_2 \Sigma x_{i2} = \Sigma y_i. \qquad (19)$$

Equation (17) can be simplified as

$$b_0 \Sigma x_{i1} + b_1 \Sigma x_{i1}^2 + b_2 \Sigma \mathrm{x}_{\mathrm{i}1} x_{i2} = \Sigma y_i x_{i1}. \qquad (20)$$

Equation (18) can be simplified as

$$b_0 \Sigma x_{i2} + b_1 \Sigma x_{i2}^2 + b_2 \Sigma x_{i2} = \Sigma y_i x_{i2}. \qquad (21)$$

If Equations (16), (17) and (18) are arranged in matrix form, we obtain

$$X'Xb = X'Y, \qquad (22)$$

with

$$\mathbf{X} = \begin{pmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} \end{pmatrix}.$$

$$\mathbf{X}' = \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_{11} & x_{21} & \dots & x_{n1} \\ x_{12} & x_{22} & \dots & x_{n2} \end{pmatrix}.$$

$$\mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix}.$$

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}.$$

If $X'X$ is not singular (in other words, $X'X$ has an inverse or its determinant is not equal to zero) then equation (22) can be written as

$$\mathbf{b} = (\mathbf{X'X})^{-1}\mathbf{X'Y}. \tag{23}$$

The matrices in Equation (23) can be simplified to

$$\mathbf{X} = \begin{pmatrix} 1 & x_{11} \\ 1 & x_{21} \\ \vdots & \vdots \\ 1 & x_{n1} \end{pmatrix}.$$

$$\mathbf{X}' = \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_{11} & x_{21} & \dots & x_{n1} \end{pmatrix}.$$

$$\mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}.$$

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}.$$

The matrix members in Equation (23) which have been simplified can be substituted based on the values in Equation (13), so that matrices are obtained

$$\mathbf{X} = \begin{pmatrix} 1 & \ln x_{11} \\ 1 & \ln x_{21} \\ \vdots & \vdots \\ 1 & \ln x_{n1} \end{pmatrix}.$$

$$\mathbf{X}' = \begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln x_{11} & \ln x_{21} & \dots & \ln x_{n1} \end{pmatrix}.$$

$$\mathbf{Y} = \begin{pmatrix} \ln[-\ln s(x_{11})] \\ \ln[-\ln s(x_{21})] \\ \vdots \\ \ln[-\ln s(x_{n1})] \end{pmatrix}.$$

Then, based on Equation (13), a linear equation is obtained

$$\ln[-\ln s(x)] = \beta_0 + \beta_1 \ln x.$$
$$y = \beta_0 + \beta_1 x_1 . y_i = \beta_0 + \beta_1 x_{i1}; \qquad i = 1, 2, \dots, n. \tag{24}$$

Then, based on Equation (24), the equations are obtained

$$\begin{aligned} y_i &= \ln[-\ln s(x_i)]. \\ x_{i1} &= \ln x_i. \\ \beta_0 &= -\alpha \ln \beta. \\ \beta_1 &= \alpha. \end{aligned} \tag{25}$$

This produces a matrix to determine the Weibull parameters using the OLS method, namely

$$\begin{aligned} \mathbf{b} &= (\mathbf{X'X})^{-1}\mathbf{X'Y}. \\ &= \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} -\alpha \ln \beta \\ \alpha \end{pmatrix}. \end{aligned} \tag{26}$$

The hypothesis test for Weibull parameter estimation is the Wald test or partial t test which is used to determine the magnitude of the influence of the independent variable partially in explaining the dependent variable (Ghozali, 2016). In other words, the hypothesis test is used to measure the relationship between $\ln(x)$ and $\ln[-\ln s(x)]$. The hypothesis used is

$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 \neq 0$$

With test statistics:

$$t = \frac{\widehat{\beta_1}}{se(\widehat{\beta_1})},$$

as well as the following decision making criteria:

1) if $t_{\text{count}} < t_{\text{table}}$ and p-value is greater than the real level (a), then H_0 is accepted. This means that the parameters are not suitable for use in calculating mortality tables;
2) if $t_{\text{count}} > t_{\text{table}}$ and p-value is smaller than the real level (a), then H_0 is rejected. This means that the parameters are suitable for use in calculating mortality tables

3. Results and Discussion

3.1 Indonesian Mortality Table 2019

Indonesian Mortality Table 2019 divided into the Indonesian Mortality Table 2019 for men (Table 1) and the Indonesian Mortality Table 2019 for women (Table 2).

Table 1. Indonesian Mortality Table 2019 for men

x	q_x	p_x	l_x	d_x
0	0.00524	0.99476	100,000	524
1	0.00053	0.99947	99,476	53
2	0.00042	0.99958	99,423	42
…	…	…	…	…
109	0.52467	0.44267	10	5
110	0.59244	0.40756	5	3
111	1	0	2	2

Table 2. Indonesian Mortality Table 2019 for women

x	q_x	p_x	l_x	d_x
0	0.00266	0.99734	100,000	266
1	0.00041	0.99959	99,734	41
2	0.00031	0.99969	99,693	31
…	…	…	…	…
109	0.54477	0.45523	81	44
110	0.58702	0.41298	37	22
111	1	0	15	15

In the mortality table, l_x shows the number of people living at exactly x years of age. The probability that a person who is currently x years old will still be alive at x+1 years is denoted by p_x, i.e

$$p_x = \frac{l_{x+1}}{l_x},$$

On the other hand, the probability that a person dies before reaching the age of $x + 1$ years is denoted by q_x, and is formulated as

$$\begin{aligned} q_x &= \frac{d_x}{l_x}. \\ &= 1 - p_x. \end{aligned}$$

3.2 Weibull Parameter Estimation

Parameter estimation is carried out using Equations (13), (23) and (26) to find Weibull parameter values using the OLS method. The mortality model was also formed using adjustments by starting observations at ages 62 to

111. Based on the values in the Indonesian Mortality Table 2019 for men, the Weibull distribution parameter estimation results are

$$
\begin{aligned}
\boldsymbol{\beta} &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \\
&= \left(\begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln x_{11} & \ln x_{21} & \dots & \ln x_{n1} \end{pmatrix}\begin{pmatrix} 1 & \ln x_{11} \\ 1 & \ln x_{21} \\ \vdots & \vdots \\ 1 & \ln x_{n1} \end{pmatrix}\right)^{-1} \\
&\times \begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln x_{11} & \ln x_{21} & \dots & \ln x_{n1} \end{pmatrix}\begin{pmatrix} \ln[-\ln s(x_{11})] \\ \ln[-\ln s(x_{21})] \\ \vdots \\ \ln[-\ln s(x_{n1})] \end{pmatrix}. \\
&= \left(\begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln(62) & \ln(63) & \dots & \ln(111) \end{pmatrix}\begin{pmatrix} 1 & \ln(62) \\ 1 & \ln(63) \\ \vdots & \vdots \\ 1 & \ln(111) \end{pmatrix}\right)^{-1} \\
&\times \begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln(62) & \ln(63) & \dots & \ln(111) \end{pmatrix}\begin{pmatrix} \ln[-\ln s(62)] \\ \ln[-\ln s(63)] \\ \vdots \\ \ln[-\ln s(111)] \end{pmatrix}. \\
&= \left(\begin{pmatrix} 1 & 1 & \dots & 1 \\ 4{,}127 & 4{,}143 & \dots & 4{,}710 \end{pmatrix}\begin{pmatrix} 1 & 4{,}127 \\ 1 & 4{,}143 \\ \vdots & \vdots \\ 1 & 4{,}710 \end{pmatrix}\right)^{-1} \\
&\times \begin{pmatrix} 1 & 1 & \dots & 1 \\ 4{,}127 & 4{,}143 & \dots & 4{,}710 \end{pmatrix}\begin{pmatrix} -1{,}880 \\ -1{,}812 \\ \vdots \\ 2{,}388 \end{pmatrix}. \\
&= \begin{pmatrix} -34{,}509 \\ 7{,}805 \end{pmatrix}.
\end{aligned}
$$

By using the R program to calculate the OLS for the Indonesian Mortality Table 2019 for men, the values shown in Table 3 were obtained.

Table 3. OLS calculation results based on the Indonesian Mortality Table 2019 for men

Parameter	Estimation	Standard error	t-value	p-value	Adj. R-squared
β_0	-34.5988	0.8384	-41.27	2×10^{-16}	0.9722
β_1	7.8045	0.1884	41.41	2×10^{-16}	

Based on Table 3 and Equation (24), we obtain the equation

$$\ln[-\ln s(x)] = 34{,}5988 + 7{,}58045 \ln(x),$$

so with the hypothesis

$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 \neq 0$$

the decision taken is to reject H_0. This decision was taken because the probability value of the variable β_1, namely 2×10^{-16}, is smaller than $\alpha = 0.05$ which is the real level. With a real level of 0.05, it can be concluded that the parameters are suitable for use in calculating mortality tables. Based on the adjusted R-squared value, 97.22% of the data can be explained by the mortality model.

Thus, the Weibull distribution parameters in the Indonesian Mortality Table 2019 for men are

$$\alpha = 7.8045.$$
$$\beta = \exp\left(\frac{-34.5988}{-7.8045}\right).$$

$$= 84.19924736.$$

In the same way, Weibull parameters can be estimated for the 2019 Indonesian Mortality Table for women. The mortality model was formed using adjustments by starting observations at the age of 62. Thus, the results of the Weibull distribution parameter estimation for the 2019 Indonesian Mortality Table for women are

$$\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}.$$

$$= \left(\begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln x_{11} & \ln x_{21} & \dots & \ln x_{n1} \end{pmatrix} \begin{pmatrix} 1 & \ln x_{11} \\ 1 & \ln x_{21} \\ \vdots & \vdots \\ 1 & \ln x_{n1} \end{pmatrix}\right)^{-1}$$

$$\times \begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln x_{11} & \ln x_{21} & \dots & \ln x_{n1} \end{pmatrix} \begin{pmatrix} \ln[-\ln s(x_{11})] \\ \ln[-\ln s(x_{21})] \\ \vdots \\ \ln[-\ln s(x_{n1})] \end{pmatrix}.$$

$$= \left(\begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln(62) & \ln(63) & \dots & \ln(111) \end{pmatrix} \begin{pmatrix} 1 & \ln(62) \\ 1 & \ln(63) \\ \vdots & \vdots \\ 1 & \ln(111) \end{pmatrix}\right)^{-1}$$

$$\times \begin{pmatrix} 1 & 1 & \dots & 1 \\ \ln(62) & \ln(63) & \dots & \ln(111) \end{pmatrix} \begin{pmatrix} \ln[-\ln s(62)] \\ \ln[-\ln s(63)] \\ \vdots \\ \ln[-\ln s(111)] \end{pmatrix}.$$

$$= \left(\begin{pmatrix} 1 & 1 & \dots & 1 \\ 4{,}127 & 4{,}143 & \dots & 4{,}710 \end{pmatrix} \begin{pmatrix} 1 & 4{,}127 \\ 1 & 4{,}143 \\ \vdots & \vdots \\ 1 & 4{,}710 \end{pmatrix}\right)^{-1}$$

$$\times \begin{pmatrix} 1 & 1 & \dots & 1 \\ 4{,}127 & 4{,}143 & \dots & 4{,}710 \end{pmatrix} \begin{pmatrix} -2{,}325 \\ -2{,}251 \\ \vdots \\ 2{,}173 \end{pmatrix}.$$

$$= \begin{pmatrix} -35.4544 \\ 7.9187 \end{pmatrix}.$$

By using the R program to calculate the OLS Indonesian Mortality Table 2019 for women, the values obtained are as shown in Table 4.

Table 4. OLS calculation results based on the Indonesian Mortality Table for women

Parameter	Estimation	Standard error	t-value	p-value	Adj. R-squared
β_0	-35.4544	0.7661	-46.28	2×10^{-16}	0.9773
β_1	7.9187	0.1722	45.99	2×10^{-16}	

Based on Table 4 and Equation (24), we obtain the equation

$$\ln[-\ln s(x)] = 35.4544 + 7.9187 \ln(x)$$

so with the hypothesis

$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 \neq 0$$

the decision taken is to reject H_0. This decision was taken because the probability value of the variable β_1, namely 2×10^{-16}, is smaller than $\alpha = 0{,}05$ which is the real level. With a real level of 0.05, it can be concluded that the parameters are suitable for use in calculating mortality tables. Based on the adjusted R-squared value, 97.33% of the data can be explained by the mortality model.

Thus, the Weibull distribution parameters in the Indonesian Mortality Table 2019 for women are

$$\alpha = 7.9187.$$

$$\beta = \exp\left(\frac{-35.4544}{-7.9187}\right).$$
$$= 87.99681028.$$

3.3 Weibull Mortality Table

After estimating the Weibull parameters based on the Indonesian Mortality Table2019, a Weibull Mortality Table can be created for men and women. The table will show mortality values such as the Indonesian Mortality Table 2019. The chance of living and the chance of dying $<x>$ at age $x+1$ year are calculated based on Equations (22) and (23) respectively. Then, the number of $<x>$ who live and the number $<x>$ who die before reaching age $x+1$ can be calculated.

The male Weibull Mortality Table obtained is provided in Appendix 1. Based on Appendix 1, the highest age in the male Weibull Mortality Table is 115 years. Then, someone who is 114 years old has a 52.8% chance of dying before reaching that highest age.

The female Weibull Mortality Table, which is created in the same way as the male Weibull Mortality Table. The female mortality table is available in Appendix 2. Based on Appendix 2, the highest age in the Weibull Mortality Table for females is 119 years. A 118-year-old person has a 50.6% chance of dying before reaching the next age (in other words, the highest age in the Weibull Mortality Table).

4. Conclussion

The Weibull Mortality Table which was created using the Weibull mortality law and based on the 2019 Indonesian Mortality Table has several differences with the 2019 Indonesian Mortality Table. In the Weibull Mortality Table for men, the highest age is 115. This age is 4 years older than the highest age in the Indonesian Mortality Table 2019 male, namely 111 years. Then, the highest age in the Weibull Mortality Table for women is 119 years, 8 years older than the highest age in the 2019 Indonesian Mortality Table for women, namely 111 years.

References

Anderson, M. C. (2004). International Conflict: An analysis of information, duration and reccurence.New York University.

Chen, L. (2010). Analysis of Joint Life Insurance with Stochastic Interest Rates.

Ghozali, I. (2016). Multivariate Analysis Application with IBM SPSS 23 program. Semarang: Diponegoro University Publishing Agency, 4, 352.

Kleinbaum, D. G., dan Klein, M. (2012). Survival Analysis: A Self-Learning Text. New York: Springer.

Mehr, R. I., dan Gustavson, S. G. (1987). Life Insurance Theory and Practice. Texas: Business Publication, Inc.

Suryani, S. L., Ruswandi, R., & Faisol, A. (2020). The Annual Premium of Life Insurance on the Joint-Life Status Based on the 2011 Indonesian Mortality Table. Decimal: Journal Mathematics. 3(3), 263-270

Nelso, J. A. S. (2002).Framing and Levels of Abstraction in Economic Theory with an Application to Public Land Management. The University of Utah.