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Construction of the Mortality Table with Gompertz's Law Using the 2019 TMI Reference

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Abstract

The Indonesian Mortality Table, which was specifically based on the real conditions of Indonesia, was successfully created for the first time in 1993. The mortality table was called the Indonesian Mortality Table (IMT) I of 1993. In its preparation from IMT II to IMT III, the time span was long enough that the Asosiasi Asuransi Jiwa Indonesia (AAJI) and Persatuan Aktuaris Indonesia (PAI) formed a team to prepare the Indonesian Mortality Table IV (IMT IV) which studied data during the study period between 2013-2017. Finally, the IV Indonesian Mortality Table was compiled in 2019. In this study, the mortality table used was the 2019 Indonesian Mortality Table issued by Persatuan Aktuaris Indonesia (PAI), as well as a modification of the 2019 IMT with Gompertz Law. In Gompertz's Law, one method that can be used is the least power square method. This study will show the suitability of Gompertz's Law against TMI 2019 for men and TMI 2019 for women, as well as to determine the suitability of Gompertz's Law using the Least Square Method for IMT 2019 for men and IMT 2019 for women.

Keywords: Gompertz, Least square method, Indonesia Mortality Table

1. Introduction

The mortality table which specifically originates from the real conditions of Indonesia was successfully prepared for the first time in 1993. The mortality table was called the Indonesian Mortality Table (TMI) I of 1993. Then it was continued with the successful compilation of TMI II of 1999. After 12 years (more than 1 decade) was just produced by TMI III in 2011. The table was made with data between 2004-2008. Seeing the long time span between the preparation of TMI II and TMI III, the Indonesian Life Insurance Association (AAJI) and the Association of Indonesian Actuaries (PAI) formed a TMI IV drafting team to study data during the study period between 2013-2017. The result is TMI IV 2019.

One approach that can be used to calculate male and female mortality is the Gompertz mortality law. Gompertz's law only takes into account the death factor, so that it fits the character of the mortality table whose elements are constructed based on reducing the number of population by cause of death (Willemse & Kaas, 2007). Many things can cause a person's death such as accidents, disease, natural disasters, and the age factor. In this case, Gompertz's law only takes into account a person's death due to age.

In this article, Gompertz's mortality law is used to build a mortality table using TMI IV 2019 as reference data. Furthermore, this research reviews the suitability between the resulting mortality table and the mortality table that is the reference, namely TMI IV 2019.

Several studies regarding the use of the Gompertz mortality law to produce mortality tables were carried out, among others, by Makeham, (1867)., Turoń & Kubik (2020). Mitus (2016), has conducted a suitability analysis between the mortality tables constructed with Gompertz's law for TMI III of 2011 for male and female gender. Makeham (1867). research produced a mortality table with Gompertz's law which contains the number of people living at the age of x years (lx), the number of people who died between the ages of x and $x+1$ year (dx), the probability that a person who is currently x years old will survive to age $x+1$ year (p_x), and the probability that a person currently aged x years will die before reaching age $x + 1$ year (q_x). Analysis of the probability of dying q_x, produces an exponential graph of the function q_x.

Furthermore, Turoń, & Kubik, (2020), calculated the size of the tabarru fund using the cost of insurance (COI) method using a mortality table built using the Gompertz mortality law. In his research, Turoń, & Kubik, (2020), constructed a mortality table with reference to the 2011 TMI. The simulations carried out using several assumptions on the level of investment and management costs, concluded that the increasing age of the insurance participants required the insurance company to allocate a larger percentage of tabarru funds. This is because a person's chance of dying will increase with age.

According to Robine (2001). also analyzes the mortality law that was constructed using the Gompertz mortality law using the 2011 TMI reference. The estimation of the parameters in Gompertz's law is carried out using the maximum likelihood estimation method. Based on the average relative error (ARE) calculated to estimate the probability of surviving P_x, the 2011 TMI results for men and 2011 TMI for women are more suitable when approached with the Gompertz mortality law compared to the use of Makeham's mortality law. The agreement occurred at the following suggested age intervals: (1) for women 0 - 10 years, 10 - 20 years, and 70 - 80 years; (2) for males at age intervals 0 - 10 years, 10 - 20 years, 20 - 30 years, and 60 - 70 years. The recommended age interval is taken based on the mean absolute percentage error (MAPE) below 1% for the estimation of the probability of surviving p_x. As a result, at that age interval, the estimated value of P_x is very close to the P_x value in the 2011 TMI.

According to Handoyo, et al., (2019) in his analysis regarding the calculation of the annual premium value of dual-purpose combined life insurance using the Experimental method. The research used in the illustration of a life insurance policy case involving two insureds with insurance paid if one of the insureds dies. Based on the results of the discussion, the probability of survival under the Gompertz assumption is smaller when compared to the data on the probability of survival from the 2011 Indonesian Mortality Table. This has the effect of differences in the value of combined endowment life insurance premiums. In addition, the influence of exchange rates on interest rates also affects the value of combined life insurance annuities.

According to Willemse & Koppelaar, (2000), in her analysis discussing endowment contingent life insurance premiums, Gompertz's law is used. It can be concluded that the annual contingent endowment life insurance premium based on Gompertz law depends on the Gompertz distribution constant, the cash value of the annuity and the probability of conditional death, that is, the condition is that participants aged *x* years die and participants aged y years die. The cash value of the annuity depends on the discount factor and the probability of life where the probability of survival according to Gompertz's law is less than the probability of survival on the mortality table (Milevsky, 2005). Furthermore, the annual premium for joint life contingent endowment insurance is greater than the annual premium for last survivor endowment contingent life insurance.

According to Cohen, et al., (2018) in his analysis which aims to determine the annual dual-purpose life insurance premium using De Moivre's law and Gompertz's law, the dual-purpose life insurance premium with Gompertz's law is higher than the dual-purpose life insurance premium with De Moivre's law. In Mr. Rano's case, the longer the coverage period, the smaller the premium to be paid, the higher the age of the insured when taking insurance, the greater the premium to be paid and the greater the expected compensation, the greater the premium to be paid.

According to Lee, et al., (2014), in his research discussing the effect of using the Gompertz mortality law on determining the amount of endowment life insurance using the Full Preliminary Term method. It can be concluded that the reserve amount at the end of coverage without the Gompertz mortality law and based on the Gompertz mortality law with the Full Preliminary Term method is of the same value. Therefore, the insurance company can pay the sum assured according to the agreement. The effect of the calculation of reserves based on the Gompertz mortality law is that the greater the age of the insured, the amount of reserves tends to decrease because there is a Gompertz constant which affects the amount of reserves. If the insured makes a claim in the middle of the coverage period, in the same year and at the same age, the calculation of reserves using the Full Preliminary Term method without the Gompertz mortality law gives better results every year.

According to Keeler, et al., (2010), in his analysis which discusses the calculation of tabarru funds using the Mahekam mortality law and Gompertz mortality law using the Cost of Insurance method. The calculation of tabarru funds obtained by applying the Gompertz and Makeham mortality law to the Cost of Insurance (COI) method has a difference because there is parameter A which states the factors that take into account death other than age. In addition, factors that influence the calculation of tabarru funds using the COI method include gender, level of investment and the amount of management fees. For example, the higher the investment, the lower the tabarru fund percentage.

Referring to the description above, in this study a mortality table was prepared using Gompertz's law using the 2019 TMI basis. Parameter estimation in Gompertz's law was carried out using the least squares method. The purpose of this study is to construct a mortality table with Gompertz's law using the 2019 TMI reference and compare the suitability of the results graphically with the mortality table as the reference. The construction of the mortality table was carried out for male and female gender (Muzaki, et al., 2020). The difference between this study and previous studies lies in the mortality table used as a reference and the parameter estimation method. The construction results of this mortality table can be used as a basis for calculating the amount of tabarru' funds.

2. Literature Review

2.1. Mortality Table

Life table or mortality table is a table that describes the distribution of death rates in a group of people observed at certain time intervals based on age group (Aprijon, 2021). In compiling the mortality table, there are several laws that can be used, namely the Gompertz, de Moivre, Makeham, and Weibull laws. The construction of the mortality table is carried out using several mathematical equations in equations (1), (2), and (3). These equations are generally accepted, in the sense that they do not depend on the mortality law chosen or used.

$$d_x = l_x - l_{x+1} \tag{1}$$

$$p_x = \frac{l_{x+1}}{l_x} \tag{2}$$

$$q_x = \frac{l_x - l_{x+1}}{l_x} \text{ or } q_x = \frac{d_x}{l_x} \tag{3}$$

with:

l_x: the number of people living at the age of x years

d_x : the number of people who died between the ages of x and $x+1$ year

p_x : the probability that a person currently aged x years will survive to $x+1$ age

q_x : the probability that a person currently aged x years old dies before reaching age age $x+1$

The values for the variable $q_x$with $x = 0,1,2,3,\dots,112$ are calculated based on the mortality table or adjusted mortality table using the selected mortality law.

In this study, Gompertz's mortality law was used. Gompertz's law is one of the laws used to describe and explain a person's age at death. According to Gompertz's law, the death rate for an individual who is currently x years old follows the equation:

$$\mu_x = Bc^x, \qquad B > 0, \qquad c > 1, \qquad x \geq 0 \tag{4}$$

Parameter B is associated with the probability of dying, and parameter c is the increase in failure or death.

The probability functions tp_x and tq_x according to Gompertz's mortality law are:

$$_tp_x = exp\left[-\frac{Bc^x}{ln\ c}(c^t - 1)\right], \tag{5}$$

$$_tq_x = 1 - exp\left[-\frac{Bc^x}{ln\ c}(c^t - 1)\right]. \tag{6}$$

To complete the mortality table with Gompertz's law, the following equations are obtained by taking $t = 1$:

$$l_x = lo.exp\left[-\frac{Bc^x}{ln\ c}(\mathrm{c} - 1)\right], \tag{7}$$

$$p_x = exp\left[-\frac{B}{ln\ c}(c^x - 1)\right], \tag{8}$$

$$q_x = 1 - exp\left[-\frac{Bc^x}{ln\ c}(c - 1)\right] \tag{9}$$

Equation (9) is used to estimate parameters B and c using the least squares method, as follows:

$$q_x = 1 - exp\left[-\frac{Bc^x}{ln\ c}(c - 1)\right]$$

$$1 - q_x = exp\left[-\frac{Bc^x}{ln\ c}(c - 1)\right]$$

$$ln\ (1 - q_x) = -\frac{Bc^x}{ln\ c}(c - 1)$$

$$ln\ \left(\frac{1}{(1 - q_x)}\right)^{-1} = -\frac{Bc^x}{ln\ c}(c - 1)$$

$$-ln\left(\frac{1}{(1 - q_x)}\right) = -\frac{Bc^x}{ln\ c}(c - 1)$$

$$ln\ \left(\frac{1}{(1 - q_x)}\right) = \frac{Bc^x}{ln\ c}(c - 1)$$

$$ln\ \left(ln\left(\frac{1}{(1 - q_x)}\right)\right) = ln\ \left(\frac{Bc^x}{ln\ c}(c - 1)\right) \tag{10}$$

$$ln\left(ln\left(\frac{1}{(1-q_x)}\right)\right) = ln\ c^x + \ ln\left(\frac{Bc^x}{ln\ c}(c-1)\right)$$

Equation (10) is a regression equation $y_i = \beta + \ \alpha\,.x_i + \varepsilon_i$ with

$$y = ln\left(ln\left(\frac{1}{(1-q_x)}\right)\right)$$

$$\alpha = \ ln\ c$$

$$\beta = ln\left(\frac{B}{ln\ c}(c-1)\right)$$

The estimator for the regression equation regression $y_i = \beta + \ \alpha\,.x_i + \varepsilon_i$ is $y_i = \beta + \ \alpha\,.x_i + \varepsilon_i$ adalah $\widehat{y_i} = b + a\,.x_i$Based on equation (10), there are two variables whose values are available in the mortality table, namely the age of the individual q and the probability of dying for an individual aged x, namely q_x. The regression equation is obtained from the x and q_x. values found in the 2019 TMI for both sexes. By using equation (10) and placing the age variable x on the x −axis and the y-axis used for the equation

$$y = \ ln\left(ln\left(\frac{1}{(1-q_x)}\right)\right)$$

The regression equation we are looking for is produced. The process of forming the regression equation based on the data is carried out in the Results and Discussion section.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_i - A_t}{A_i}\right| \times 100\%$$

2.2. MAPE (Mean Absolute Percentage Error)

MAPE (Mean Absolute Percentage Error) is a statistical tool used to measure the accuracy of a statistical model in predicting or forecasting. MAPE Formula:

with

n: is the sample size

A_i: is the value of q_x in TMI 2019

A_t: is the value of q_x in Gompertz's Law

If the MAPE value is <10%, the more accurate or better the model is in forecasting. Meanwhile, the MAPE value is > 10%, a forecasting model is not accurate or not good.

3. Research Methods

The data analyzed in this study is available in the 2019 Indonesian Mortality Table (TMI). The construction or modification of the mortality table is carried out using Gompertz's law. Parameter estimation method on Gompertz's law is done by least squares method. Mortality table calculations are completed with the help of the Microsoft Excel application to speed up the process and improve the accuracy of the calculations.

The steps in this research are as follows:

1. determine the mathematical equations needed to produce the 2019 TMI adjusted to Gompertz's law;
2. determine the life and death chances of the insured through the mortality table;
3. determine the regression equation that corresponds to the mortality rate on Gompertz's law;
4. determine the constants (parameters) in the Gompertz mortality law using the least squares method
5. made a 2019 TMI adjustment based on Gompertz law.

4. Results And Discussion

4.1. TMI Construction 2019 Based on Gompertz Law

The death rate for Gompertz's law has been given in Equation (4). Parameter B in Equation (4) is associated with the probability of death and parameter c in Equation (4) represents the rate of failure or death. Based on equation (10) by taking the x-axis for the variable x which represents age and

$$y = ln\left(ln\left(\frac{1}{(1-q_x)}\right)\right)$$

Placed in the y-axis position, and with the help of Microsoft Excel we get regression equation for male and female. The q_x value at

$$y = ln\left(ln\left(\frac{1}{(1-q_x)}\right)\right)$$

Is taken from TMI 2019 The regression equation obtained is $\hat{y_i} = 0.07916317x - 9.20048547$for male sex and $\hat{y_i} = 0.079063545x - 9.509513076$ for female gender.
Based on the two regression equations, the values for the Gompertz parameter are $B = 0.000144661$ and $c = 1.08238092$ for male sex and $B = 0.000107444$ and $C = 1.082273092$ for female. Substitute B and C in equation (9) to get the value of q_x in the Gompertz mortality tables for boys and girls.

4.2. TMI 2019 Construction Results Based on Gompertz Law

Furthermore, based on Equation (9) the q_x values are obtained for men and women and in Table 1 a table is compiled containing q_x for men and women based on the 2019 TMI and based on Gompertz's law with reference to the 2019 TMI.

Table 1: Summary of the Probability of Death Based on the 2019 TMI and q Law with the 2019 TMI Reference

Age	Probability of Death (q_x)			
	Man (TMI 2019)	Female (TMI 2019)	Man (Gompertz)	Female (Gompertz)
0	0.00524	0.00266	0.000150530	0.000111799
1	0.00053	0.00041	0.000162929	0.000120997
2	0.00042	0.00031	0.000176351	0.000130951
3	0.00034	0.00024	0.000190877	0.000141724
4	0.00029	0.00021	0.000206600	0.000153383
.				
.				
110	0.59244	0.58702	0.597836451	0.487852281
111	1	1	1	1

4.3. Graphic Illustration for Mortality Table

A graphic illustration for the 2019 Indonesian Mortality Table (TMI) for men along with the results of the construction of the Mortality Table based on Gompertz's law can be seen in Figure 1. In Figure 1, a graph of the probability of death (died) at age x or q_x is presented for the 2019 Indonesian Mortality Table and results construction with Gompertz's Law, both for the male gender.

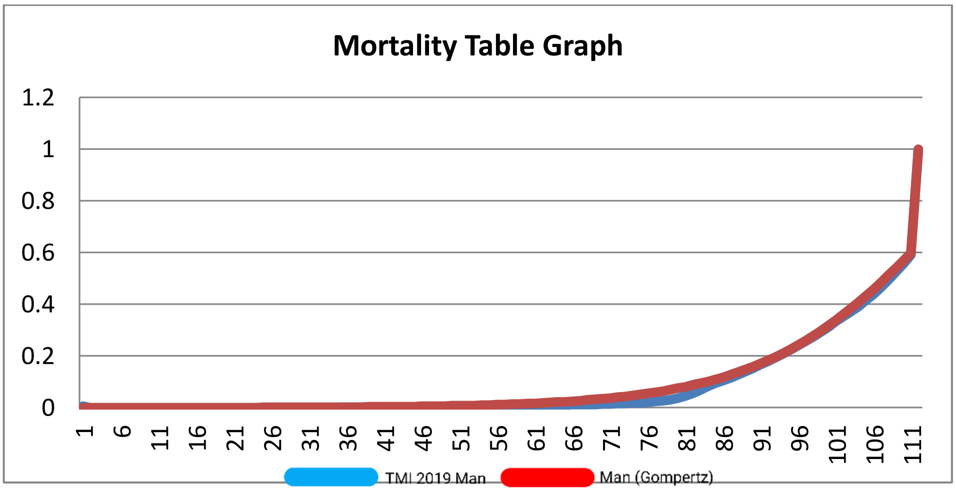

Figure 1: Graph of Death Probability for Men based on TMI 2019 and Mortality Table based on Gompertz's Law

Based on Figure 1, TMI 2019 has the lowest q_x value at the age of 10 to 12 years, namely 0.00019. Meanwhile, the Gompertz Mortality Table has the lowest $q_x$value at the age of 0 years, namely 0.00015053. In the Gompertz mortality table, the value of q_x or the probability of death at age *x* continues to increase with age, from 0 to 111 years. Whereas for TMI 2019 there is an increase and decrease in the value of q_x or the probability of death at age x, but at the age of 8 years the value of $q_x$continues to increase as you get older, up to 111 years.

A graphic illustration for the 2019 Indonesian Mortality Table for women along with the results of the construction of the Gompertz Mortality Table can be seen in Figure 2. Figure 2 presents a graph of the probability of dying at age *x* or q_x for the 2019 Indonesian Mortality Table for women and the construction results with Gompertz's law. The graph of the q_x function obtained is in exponential form, as produced in Mitus' research (2016).

By looking at Figure 2, it can be seen that TMI 2019 has the lowest q_x at the age of 11, namely 0.00018. Meanwhile, the Gompertz Mortality Table has the lowest $q_x$value at the age of 0 years, namely 0.000111799. In the Gompertz mortality table, the value of q_x or the chance of death at age x continues to increase with age, from 0 to 111 years. Whereas for TMI 2019 there is an increase and decrease in the value of q_x or the probability of death at age x, but at age 12 the value of q_x continues to increase as you get older, up to 111 years.

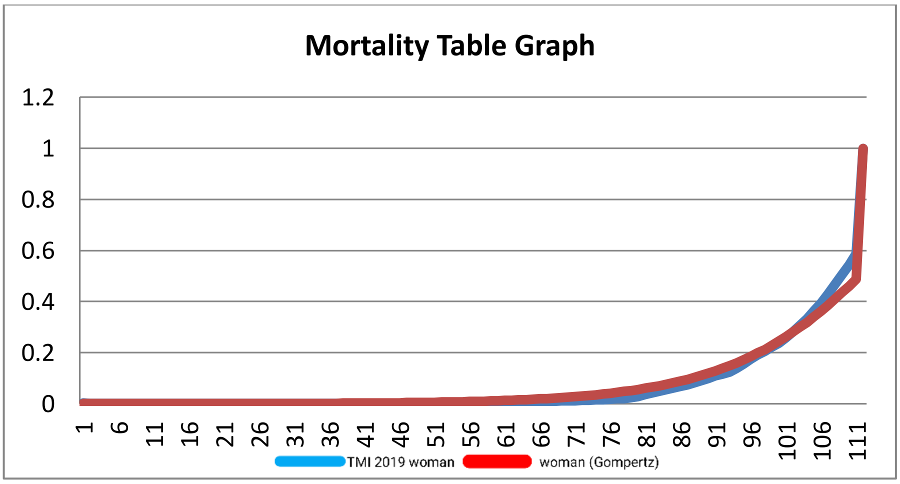

Figure 2: Graph of Death Probability for Women based on TMI 2019 and Mortality Table based on Gompertz's Law

From the MAPE results, the value of MAPE = 73.32346 is > 10%, so forecasting using Gompertz's law is not accurate or not good enough.

5. Conclusion

Based on the results of the discussion regarding the construction of the mortality table using Gompertz's law, it produces parameter values $B = 0.000144661$ and $C = 1.08238092$. The graph of the q_x function obtained is in exponential form, as produced in Mitus' research (2016). The construction results of the q_x value in the 2019 TMI table for men using Gompertz's law tend to be lower when compared to the q_x value in the 2019 TMI table for men.

Meanwhile, the construction of the q_x value in the 2019 TMI table for women with Gompertz's law gives results $B = 0.000107444$ and $C = 1.082273092$. The results of constructing the q_x value in the 2019 TMI for women using Gompertz's law tend to be lower when compared to the q_x value in the 2019 TMI for women. The graph of the function qx obtained is also in exponential form.

Thus the construction of the 2019 TMI using Gompertz's law can be said to be successful, as evidenced by the $q_x$value obtained in table 1. But forecasting using Gompertz's law is inaccurate or not good based on the MAPE value obtained, which is > 10%. So the accurate q_x value is q_x TMI 2019.

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