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Bayes Estimator for Dagum Distribution Parameters Using Non-Informative Prior Rules with K-Loss Function and Entropy Loss Function

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Abstract

The parameter estimator discussed is the p parameter estimator of the Dagum distribution with the K-loss function and the entropy loss function using the Bayes method. To get the Bayes estimator from the scale parameter of the Dagum distribution, the Jeffrey non-informative prior distribution is used based on the maximum likelihood function and the loss function for the K-loss function and the entropy loss function to obtain an efficient estimator. Determination of the best estimator is done by comparing the variance values generated from each estimator. An estimator that uses the entropy loss function is the best method for estimating the parameters of the Dagum distribution of the data population with efficient conditions met.

Keywords: Parameter estimator, Dagum distribution, Bayes method, Jeffrey prior, loss function.

1. Introduction

Inference statistics are statistics used to estimate the parameters of a population through analysis of data that has been collected from the population. After obtaining estimates of the parameters, conclusions about the population are drawn. Two important problems in statistical inference are estimator and hypothesis tester. Parameter estimates can be obtained using two methods, that is the frequency statistical method and the Bayesian statistical method. The statistical methods commonly used to estimate parameters are the moment method, maximum likelihood method and Bayes method (Mood et al., 1974).

Ahmed et al. (2013) explain that in the Bayes method, parameters are treated as random variables and the data is fixed. This parameter is viewed as a variable and its values are expressed in a distribution called the prior distribution. Priors can be compared from various angles, for example in terms of information content which can be divided into informative priors and non-informative priors. Non-informative prior distributions include Jeffrey priors (N. H. Al-Noor & Awi, 2015). Based on Bayes' theorem, the likelihood function and prior distribution are combined to form a posterior distribution.

Bain & Englehardt (1992) explain that in Bayesian statistics, if the loss function used is a k-loss function and the entropy loss function the Bayes estimator is obtained as the expectation of the posterior distribution. There are two types of estimator evaluation criteria, namely biased and unbiased. If the estimator is unbiased, then the efficient estimator is the estimator that has minimum variance and if the estimator is biased, then the efficient estimator is the estimator that has minimum mean squared error, abbreviated as MSE (Jan Gerhard & Norstrom, 1996). The way to determine an efficient estimator is through relative efficiency (Ramachandra & Tsokos, 2009).

The Dagum distribution was proposed by Camilo Dagum in the 1970s. The Dagum distribution is very useful for representing income distribution, actuarial, meteorological data as well as for survival analysis. The Dagum Distribution is a special case of the Generalized Beta II Distribution with parameters (a, b, p) or the parameters in the Generalized Beta II Distribution, namely $q = 1$ (Naqash et al, 2017). In this research, it is discussed to compare Dagum distribution scale parameter estimators using non-informative prior rules and the loss function K-loss Function and Entropy Loss Function.

2. Materials and Methods

Definition 2.1 [Bain & Englehardt, 1992] The likelihood function is a joint probability density function of n random variables $X_1, X_2, \dots, X_n$ and is expressed in the form $f(x_1, x_2, \dots, x_n; \theta)$. If $X_1, X_2, \dots, X_n$ represent a random sample from $f(x; \theta)$, then

$$L(\theta) = f(x_1;\theta)f(x_2;\theta)\ldots f(x_n;\theta),$$
$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta). \quad (1)$$

Definition 2.2 [Bain & Englehardt, 1992] Let X be a continuous random variable with a density function $f_X(x)$, For example, $Y = u(X)$ is a one-to-one transformation from $A = \{x|\, f\_X\,(x) > 0\}$ at $B = \{y|\, f\_Y\,(y) > 0\}$ with inverse $x = w(y)$. If the derivative $\frac{d}{dy}w(y)$ is continuous and not 0 at B, then the density function Y is

$$f_Y(y) = f_X(w(y))\left|\frac{d}{dy}w(y)\right|, \qquad y \in B. \quad (2)$$

An important concept in probability theory is the expectation of random variables. Below are given concepts related to expectations of random variables in the form of definitions and theorems.

Definition 2.3 [Bain & Englehardt, 1992] If X is a continuous random variable with a probability density function $f(x)$, then the expected value of X is defined by,

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx. \quad (3)$$

Variance in statistics is used for descriptive statistics, statistical inference, hypothesis testing, and sampling. Variance is an important tool in science, where statistical analysis of data is commonly used.

Definition 2.4 [Ramachandra & Tsokos, 2020] Let X be a random variable, with $E(X) = \mu$. The variance of X is denoted by $\mathrm{Var}(X)$with,

$$Var(X) = E(X-\mu)^2 \quad (4)$$

The Dagum distribution is a special form of the Generalized Beta distribution with three parameters so that it can:

$$f(x;a,b,p,) = \begin{cases} \dfrac{ap\, x^{ap-1}}{b^{ap}[1+(\frac{x}{a})^{a}]^{p+1}}, x > 0, a, b, q > 0 \\ 0 \qquad\qquad\qquad\qquad , x \text{ others.} \end{cases} \quad (5)$$

The Dagum distribution is a combined distribution of the gamma distribution and the Weibull distribution. The distribution can be obtained from the generalized gamma (GG) compound distribution whose scale parameters follow the Weibull (IW) distribution (Naqash et al., 2017).

$$f_{GG}(x) = \frac{a}{\theta^{a\,p}\,\Gamma(p)} x^{a\,p-1} e^{-\left(\frac{x}{\theta}\right)^{a}}, x > 0, (a,p,\theta) > 0$$

and,

$$f_{IW}(\theta) = \frac{a}{b^{a+1}}\left(\frac{b}{\theta}\right)^{a+1} e^{-\left(\frac{b}{\theta}\right)^{a}}; \theta > 0, a, b > 0,$$

So, the resulting mixed distribution of x is conditional on θ to obtain θ

$$f_D(x) = \int_0^{\infty} f_{GG}(x|\theta) f_{IW}(\theta) d\theta,$$
$$= \frac{a^2 b^a}{\Gamma(p)} x^{a\,p-1} \int_0^{\infty} \frac{1}{\theta^{a(p+1)+1}} e^{-(b^a + x^a)\frac{1}{\theta^a}} d\theta$$
$$f_D(x) = \frac{a\,p}{b^{a\,p}} x^{a\,p-1}\left[1+\left(\frac{x}{b}\right)^{a}\right]^{e^{-(p+1)}}. \quad (6)$$

The posterior distribution is obtained by combining sample information and prior information using Bayes' theorem which is then used in the inference process.

Definition 2.5 [Mood & McFarlane, 1974] A distribution of θ denoted by $\pi(\theta)$ is called a prior distribution, and a conditional distribution of θ given $x = (x_1, \ldots, x_2, \ldots, x_n)$, denoted by $\pi(\theta|x)$ is called the posterior distribution with the following probability density function

$$\pi(\theta|x) = \frac{\prod_{i=1}^{n} f(x_i|\theta)\pi(\theta)}{\int \prod_{i=1}^{n} f(x_i|\theta)\pi(\theta)d\theta}. \quad (7)$$

From equation (7), suppose $\frac{1}{\int \prod_{i=1}^{n} f(x_i|\theta)\pi(\theta)d\theta} = m$, which is a constant so that the posterior qualifies as a distribution, based on Definition 2.1 can be written as,

$$\pi(\theta|x) = m\, L(\theta)\pi(\theta),$$

or

$$\pi(\theta|x) \propto L(\theta)\pi(\theta). \quad (8)$$

where m is a constant and $I(\theta)$ is the defined Fisher information

$$I(\theta) = -nE\left[\frac{\partial^2 \ln f(x,\theta)}{\partial \theta^2}\right]. \quad (9)$$

The estimator gives a value that is different from the value of the parameter being estimated, so it is necessary to consider a loss which is a function of this difference.

The loss function consists of the K-loss function and the Entropy loss function, namely:

1. K-loss Function (KLF)

$$L(\hat{p},p) = \frac{(\hat{p}-p)^2}{\hat{p}\,p}, \quad (10)$$

The parameters of the K-loss Function in equation (10) are

$$\hat{p}_{kl} = \sqrt{\frac{E(p)}{E\left(\frac{1}{p}\right)}}, \quad (11)$$

2. Entorpy Loss Function

In many practical cases, it is more realistic to show the loss in terms of the ratio $\frac{\hat{p}}{p}$ whose asymmetric loss function is the entropy loss as follows:

$$L(\hat{p},p) = \left(\frac{\hat{p}}{p}\right) - \log\left(\frac{\hat{p}}{p}\right) - 1, \quad (12)$$

The parameters of the Entorpy Loss Function in equation (13) are

$$\hat{p}_{el} = \left[E\left(\frac{1}{p}\right)\right]^{-1}. \quad (13)$$

3. Results and Discussion

The Dagum distribution is a continuous distribution. Dagum distribution with probability density function in equation (5) which has 3 parameters, namely $a,\ p,$ and b. Identification is continued by forming a likelihood function, based on Dagum. The likelihood function of the Dagum distribution probability density function in equation (6) is for example $x_1,\ x_2, \dots, x_n$ is an independent random variable of size n, so using Definition 2.1 ie

$$L(p) = \left(\frac{a\,p}{b^{a\,p}}\right)^n \prod_{i=1}^{n} x_i^{\,a\,p-1} \prod_{i=1}^{n}\left[1+\left(\frac{x_i}{b}\right)^a\right]^{-(p+1)}, \quad (14)$$

Next, to determine the estimator, first determine the function $\ln(L(p))$ from equation (14) using the equationln$(\ L(\theta)) = 0$, namely

$$\ln(L(p)) = n(\ln a + \ln p - a\,p\ln b) + (ap-1)\sum_{i=1}^{n}\ln x_i - (p+1)\sum_{i=1}^{n}\ln\left[1+\left(\frac{x_i}{b}\right)^a\right]. \quad (15)$$

estimator p̂ obtained based on the maximum likelihood function obtained by solving the equation $\frac{\partial}{\partial\theta}\ln(\ L(\theta)) = 0$, based on the equation (15), obtained

$$\frac{\partial L}{\partial p} = \frac{n}{p} - n\,a\ln b + a\sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}\ln\left[1+\left(\frac{x_i}{b}\right)^{a}\right] = 0, \tag{16}$$

$$\frac{n}{p} = \sum_{i=1}^{n}\ln\left[1+\left(\frac{x_i}{b}\right)^{a}\right] - \sum_{i=1}^{n}\left(\ln\frac{x_i}{b}\right)^{a} = 0,$$

$$\hat{p}_{ml} = \frac{n}{\sum_{i=1}^{n}\ln\left[1+\left(\frac{x_i}{b}\right)^{-a}\right]} \tag{19}$$

The prior used to explain the parameter p of the Dagum distribution is the non-informative Jeffrey prior. Jeffrey's prior was proposed by Harold Jeffrey in 1946. The first differential of the function $\ln(L(p))$ in equation (15) is obtained, in part as follows:

$$\frac{\partial^2 L}{\partial p^2} = \frac{\partial}{\partial p}\left(\frac{n}{p}\right)\frac{\partial}{\partial p}(n\,a\ln b) + \frac{\partial}{\partial p}\left(a\sum_{i=1}^{n}\ln x_i\right)\frac{\partial}{\partial p}\sum_{i=1}^{n}\ln\left[1+\left(\frac{x_i}{n}\right)^{a}\right], \tag{20}$$

Thus, the second derivative of equation (19) is

$$\frac{\partial^2 L}{\partial p^2} = -\frac{n}{p^2}, \tag{21}$$

Below, the expected value from equation (4.6) is determined using the equation $I(p) = \pi(p) \propto [I(p)]^m$, to determine Fisher information

$$I(p) = -E\left(-\frac{\partial^2 L}{\partial p^2}\right) = \frac{n}{p^2},$$

$$\pi(p) \propto \frac{1}{p^{2m}};\, m > 0. \tag{22}$$

equation (20) which is a non-informative prior value in the one-parameter Dagum distribution. In determining the posterior distribution, the concepts of joint distribution and marginal distribution are needed. Let $\pi(p|x)$ be the posterior distribution expressed in equation (7) for example $\prod_{i=1}^{n} f(p|\theta)\pi(\theta) = p(p|x)$ and $\int f(p|\theta)\pi(\theta) = g(p|x)$. Based on equation (14), with the likelihood function equation (21) using equation (8) to get the posterior of p the posterior can be obtained

$$p(p|x)\ \alpha\ \pi(p) * L(x,p) \tag{23}$$

From equation (14) the likelihood function where $L(x,p)$ is the likelihood function of the one-parameter Dagum distribution and $\pi(p)$ is the non-informative prior value on the one-parameter Dagum distribution

$$p(p|x)\ \alpha\ p^{-2m}\, p^{n}\prod_{i=1}^{n}\frac{x_i}{b^{a\,p}}\prod_{i=1}^{n}\left[1+\left(\frac{x_i}{b}\right)^{a}\right]^{-p},$$

So it is obtained

$$p(p|x)\ \alpha\ p^{-2m}p^{n}\left[e^{\sum_{i=1}^{n}\ln\left(\frac{x_i}{b}\right)^{ap}}\right]\left[e^{\sum_{i=1}^{n}\ln\left[1+\left(\frac{x_i}{b}\right)^{a}\right]^{-p}}\right],$$

$$p(p|x)\ \alpha\ Kp^{n-2m}e^{-p\,T}\,. \tag{24}$$

T is equation (17) and K is a normal constant. The next step is to determine $g(p|x)$. Using equation (4.9), we obtain

$$g(p|x) = \int_0^\infty p(p|x),$$

where, $pT = x$

$dx = Tdp$

$$g(p|x) = \int_0^\infty K\left(\frac{x}{T}\right)^{n-2m} e^{-x}\frac{dx}{T},$$

$$g(p|x) = K\frac{\Gamma(\mathrm{n}-2\mathrm{m}+1)}{\mathrm{T}^{n-2m+1}}\,. \tag{25}$$

By substituting equation (22) and equation (23) into the posterior distribution in equation (7), we obtain

$$\pi(p|x) = \frac{\mathrm{T}^{n-2m+1}}{\Gamma(n-2m+1)} p^{n-2m} e^{-pT}. \tag{26}$$

Equation (24) is a gamma probability density function with parameter T, with $\beta = n - 2m + 1$. The Bayes estimator uses the K-loss function and the Entropy loss function, namely:

(a) ***K-Loss Function***

The Bayes parameter uses the k-loss function in equation (11), and using Definition 2.3, the $E(p|x)$ value is obtained

$$E(p|x) = \int_0^\infty p\, \pi*(p|x) dp,$$

$$E(p|x) = \int_0^\infty p \frac{T^\beta}{\Gamma\beta} p^{\beta-1} e^{-pT} dp,$$

$$E(p|x) = \frac{\beta}{T} \cdot$$

With value, $E(p^{-1}|x)$

$$E(p^{-1}|x) = \int_0^\infty p^{-1} \frac{\mathrm{T}^\beta}{\Gamma\beta} p^{\beta-1} e^{-pT} dp,$$

$$= \frac{T^\beta}{\Gamma\beta} \int_0^\infty p^{\beta-2} e^{-pT} dp,$$

Based on, Definition $\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx$ can be written:

$$= \frac{T^\beta}{\Gamma\beta} \frac{\Gamma(\beta-1)}{T^{\beta-1}},$$

$$E(p^{-1}|x) = \frac{\mathrm{T}}{\beta - 1} \cdot$$

Then, Bayes parameters can be obtained

$$\hat{p}_{kl} = \sqrt{\frac{E(p|x)}{E(p^{-1}|x)}} = \frac{\sqrt{\beta(\beta-1)}}{T} \cdot \tag{27}$$

(b) ***Entropy Loss Function***

The Bayes parameter uses the Entropy Loss Function in equation (13), as follows, using Definition 2.3 to obtain the $\left[E\left(\frac{1}{p}\right)\right]^{-1}$ value,

$$\left[E\left(\frac{1}{p}\right)\right]^{-1} = \int_0^\infty p\, \pi*(p|x) dp,$$

$$= \left[\int_0^\infty p^{-1} \frac{\mathrm{T}^\beta}{\Gamma\beta} p^{\beta-1} e^{-pT} dp\right]^{-1},$$

$$\left[E\left(\frac{1}{p}\right)\right]^{-1} = \frac{\beta - 1}{T},$$

Then, Bayes parameters can be obtained

$$\hat{p}_{el} = \left[E\left(\frac{1}{p}\right)\right]^{-1} = \frac{\beta - 1}{T}, \tag{28}$$

In determining the comparison of estimators, the first step is to determine the expectation and variance, where is $T = \sum_{i=1}^n \ln\left[1 + \left(\frac{x}{b}\right)^{-a}\right]$ a nonlinear expected function, then transformation method is necessary. Where X is the Dagum distribution with parameters (b, a, p), so T in equation (17) has a gamma distribution with parameters (n, p). $T = \sum_{i=1}^n \ln\left[1 + \left(\frac{x}{b}\right)^{-a}\right]$. Next to Definition 2.2, we get the inverse $\mathrm{T} = \ln\left[1 + \left(\frac{x}{b}\right)^{-a}\right]$,

Using calculus $e^{\ln x} = x$ can get

$$e^t = 1 + \left(\frac{x}{b}\right)^{-a},$$

$x = b\,(e^t - 1)^{-\frac{1}{a}},$
The derivatives of x is

$$\frac{dx}{dy} = -\frac{b}{a}\,(e^t - 1)^{-\frac{1}{a}-1}e^t, \tag{29}$$

Next, to determine the estimator, previously carried out the transformation method, by substituting equation (15) and equation (27) into Definition 2.2 to obtain,

$$g_T(t) = \frac{ap}{b^{a\,p}}\left(b\,(e^t - 1)^{-\frac{1}{a}}\right)^{ap-1}\left[1 + \left(\frac{b\,(e^t - 1)^{-\frac{1}{a}}}{b}\right)^{a}\right]^{-(p+1)}\left|-\frac{b}{a}\,(e^t - 1)^{-\frac{1}{a}-1}e^t\right|,$$

$$= \frac{ap}{b^{a\,p}}\left(b\,(e^t - 1)^{-\frac{1}{a}}\right)^{ap-1}\,[1 + (e^t - 1)^{-1}]^{-(p+1)}\,\frac{b}{a}\,(e^t - 1)^{-\frac{1}{a}-1}e^t,$$

$$g_T(t) = p\,e^{-pt}. \tag{30}$$

Thus from equation 28 it is obtained,

$$g_T(t) = \frac{p^n}{\Gamma(n)}e^{-pt}t^{n-1}\,; t > 0, p > 0. \tag{31}$$

From equation 29 we obtain the expected gamma distribution, namely:

$$E(t^{-r}) = \frac{p^n}{\Gamma(n)}\int_0^\infty e^{-pt}t^{n-r-1}\,dt,$$

Furthermore,

$$E(t^{-r}) = \frac{p^n}{\Gamma(n)}\int_0^\infty e^{-T}\left(\frac{T}{p}\right)^{n-r-1}\frac{dT}{p},$$

$$E(t^{-r}) = p^r\,\frac{\Gamma(n-r)}{\Gamma(n)}, \tag{32}$$

If $r = 1$, then

$$E(t^{-1}) = \frac{p}{(n-1)}.$$

If $r = 2$, then

$$E(t^{-2}) = \frac{p^2}{(n-1)(n-2)}.$$

Then $var(t^{-1}) = E(t^{-1})^2 - [E(t^{-1})]^2$

$$var(t^{-1}) = \frac{p^2}{(n-1)(n-2)} - \frac{p^2}{(n-1)^2} = \frac{p^2}{(n-1)(n-2)}. \tag{33}$$

Next, to obtain the variance parameters $\hat{p}_{ml}, \hat{p}_{kl}\,\hat{p}_{el}$ are searched using equation (31) which,

$$E(\hat{p}_{ml}) = \frac{n}{T} \text{ then } V(\hat{p}_{ml}) = \frac{n^2p^2}{(n-1)\,(n-2)}. \tag{34}$$

$$E(\hat{p}_{kl}) = \frac{\sqrt{\beta(\beta-1)}}{T} \text{ then } V(\hat{p}_{kl}) = \frac{\beta(\beta-1)p^2}{(n-1)\,(n-2)}. \tag{35}$$

$$E(\hat{p}_{el}) = \frac{\beta-1}{T} \text{ then } V(\hat{p}_{el}) = \frac{(\beta-1)^2p^2}{(n-1)\,(n-2)}. \tag{36}$$

By using equation (32), equation (33) and equation (34) it is obtained that the parameter estimator is more efficient than the $\hat{p}_{el}$ parameter estimator if

$$\hat{p}_{el} < \hat{p}_{kl} < \hat{p}_{ml},$$

Next, to determine an efficient estimator between the 3 estimators, relative efficiency needs to be carried out. Of the three estimators $\hat{p}_{ml}, \hat{p}_{kl}\,\hat{p}_{el}$

For the possibility that the parameter estimator $\hat{p}_{kl}$ is more efficient than the parameter estimator $\hat{p}_{ml}$

$$e_1 = \frac{V(\hat{p}_{ml})}{V(\hat{p}_{kl})} = \frac{n^2p^2}{(n-1)\,(n-2)} = \frac{n^2}{\beta(\beta-1)}.$$

Parameter estimator $\hat{p}_{kl}$ is more efficient than the parameter estimator $\hat{p}_{ml}$.
Next, For the possibility that the parameter estimator $\hat{p}_{el}$ is more efficient than the parameter estimator $\hat{p}_{kl}$,

$$e_2 = \frac{V(\hat{p}_{kl})}{V(\hat{p}_{el})}, = \frac{\beta(\beta-1)p^2}{(n-1)\,(n-2)} = \frac{\beta}{(\beta-1)}.$$

Parameter estimator $\hat{p}_{el}$ is more efficient than the parameter estimator $\hat{p}_{kl}$.
Next, For the possibility that the parameter estimator $\hat{p}_{el}$ is more efficient than the parameter estimator $\hat{p}_{ml}$,

$$e_3 = \frac{V(\hat{p}_{el})}{V(\hat{p}_{ml})}, = \frac{(\beta-1)^2 p^2}{(n-1)\,(n-2)} = \frac{(\beta-1)^2}{n^2}.$$

Parameter estimator $\hat{p}_{el}$ is more efficient than the parameter estimator $\hat{p}_{ml}$.

Next, data simulations were carried out using R software. The estimators obtained using the Bayes method will be compared using simulations. Data simulation was carried out by generating various types of data conditions involving two types of p parameter values and five types of sample sizes, namely $n = 10, 25, 50, 75, 100$. Next, the variance value is calculated. The simulation in this study was carried out using the R program. For example, if it is known that the values $p = 10.95$ and $p = 6.29$ and $m = 0.5$, the Bayes estimator is obtained as shown in the Table 1.

Table 1: Parameter variance value

n	$p = 10.95$			$p = 6.29$		
	$\hat{p}_{ml}$	$\hat{p}_{kl}$	$\hat{p}_{el}$	$\hat{p}_{ml}$	$\hat{p}_{kl}$	$\hat{p}_{el}$
10	18.50347	16.6531	14.9878	6.10557	5.49501	4.94551
25	5.65663	5.43036	5.21315	1.86651	1.79185	1.72017
50	2.60096	2.54894	2.49796	0.85824	0.84107	0.82425
75	1.68719	1.66469	1.6425	0.55672	0.54929	0.54197
100	1.24833	1.23585	1.22349	0.41191	0.40779	0.40371

Table 1 shows the results of the variance of the parameters. It can be seen that the p value produces a smaller relative efficiency value for the Bayes Entropy loss function method compared to maximum likelihood and k-loss function. This can be seen clearly in the data in Table 1 and graphs or images can be produced in Figures 1 and Figure 2

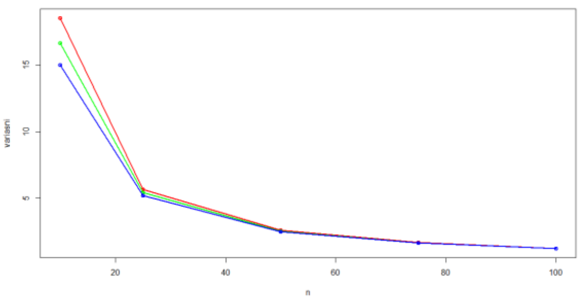

Figure 1: with parameter $p = 10.95$.

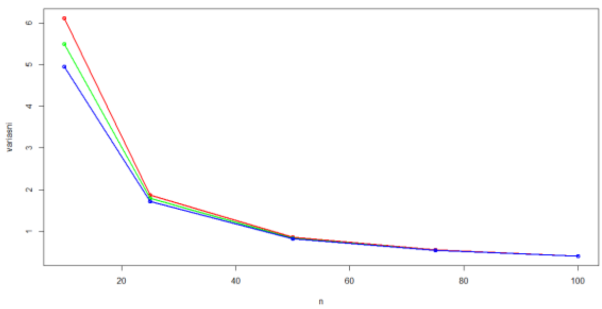

Figure 1: with parameter $p = 10.95$.

Figure 1 and Figure 2 are illustrations of the relative efficiency of the parameters $p = 10.95$ and $p = 6.29$. The highest graph in red is the maximum likelihood value, the green graph is the k-loss function, and the blue graph is the Entropy loss function. It can be seen that the value of the Bayes method shows that the variance value gets smaller with the larger the sample size. The variance value in the Bayes method with the Entropy loss function shows that the smaller the number, the more relative efficiency it has compared to the maximum likelihood and k-loss function.

4. Conclusion

Based on data simulation using an estimator obtained with the R program, obtained from the relative efficiency values in the data simulation results table, it shows that if the sample size condition is larger, the variance will be smaller. The results of the variance comparison using the Bayes method using the Entropy loss function obtained smaller numbers compared to the maximum likelihood and K-loss function. So, the best method for estimating Dagum distribution scale parameters is to use the Entropy loss function.

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