Available online at https://ejournal.corespub.com/index.php/ijmsc/index

International Journal of Mathematics, Statistics, and Computing

e-ISSN 3025-0803

Vol. 2, No. 1, pp. 16-24, 2024

Optimizing the LQ45 Stock portfolio using Piecewise Linear Function: A Case Study from an Investor's Point of View

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Abstract

In optimization problem solving, both linear and nonlinear approaches can be used, with nonlinear programming considering constraints or not. One effective method of nonlinear programming is the Piecewise Linear approach, which breaks down complex nonlinear functions into straight-line segments to make their solution easier. This method can be applied in the financial sphere, such as in stock investment. This study discusses the application of piecewise linear function in optimizing investment portfolios in Bank Jago Tbk. (ARTO), Barito Pacific Tbk. (BRPT), and Go To Gojek Tokopedia Tbk. (GOTO) stocks. The purpose of this study is to provide insight that Piecewise Linear can provide optimal solutions in managing investment portfolios by calculating the risks and returns of selected stocks. The results showed a risk with a level α as big as 0.01, the expected profit from the investment of the three stocks in the calculation period reached IDR 1,803,109. On the other hand, for investors who are more cautious and have a Level α as big as 1, the anticipated profit in the same investment is around IDR 1,275,052.

Keywords: LQ45 Stock portfolio, Piecewise Linear Function, Separable Programming, Nonlinear Programming

1. Introduction

In everyday life, humans often face problems that can be overcome through a mathematical approach, especially in the context of optimization. Optimization becomes relevant in areas such as economics, industry, engineering, and other sectors. Basically, optimization aims to find the best results, be it maximum or minimum, by formulating goal functions and overcoming existing constraints (Carissimo &; Korecki, 2023; Hiller & Lieberman, 2000).

Optimization problem solving can be done through a linear approach (Carreras & Kirchsteiger, 2023) as well as non-existentar. Although linear models are often effective, in more complicated problem situations, nonlinear models are a more suitable choice (Schoukens &; Ljung, 2019). An optimization problem is categorized as nonlinear when its functions, goals, and constraints have a nonlinear form. Nonlinear programming can be classified into two categories, namely nonlinear programming that involves constraints (constrained) and that does not involve constraints (unconstrained), which will affect the method of solving it (Lavezzi et al., 2023).

Nonlinear programming with constraints considering existing obstacles or limitations in finding optimal solutions (Salman & Al-Jilawi, 2022). In contrast, seamless nonlinear programming solves problems without considering limiting factors during the calculation process until it reaches an optimization level. Some analytical approaches that can be applied to complete nonlinear programming such as Piecewise Linear.

Piecewise linear is an approach to solving nonlinear programming problems that involves approximating nonlinear functions and using straight-line segments between discrete points. This approach breaks down complex nonlinear functions into simpler parts, so that each segment can be solved by linear optimization techniques (Alkhalifa &; Mittelmann, 2022). The advantage of this approach is its ability to handle complex problems and reduce the complexity of calculations, but its accuracy depends on how well the linear approximation represents the characteristics of actual nonlinear functions. The piecewise linear method is a method that can be used as a solution to solve optimization problems in the financial sphere such as stocks.

A stock is an investment document that records a person's ownership in a company, giving the right to earn dividends or other profit sharing that the company may give to shareholders. If a company achieves profit within a certain period, investors will get a dividend distribution that is in line with the number of shares owned by each investor in the company. Nonetheless, it is important to remember that engaging in stock investing must also be able to accept risks that can lead to the loss of significant amounts of funds. To reduce these risks, investors need to constantly monitor stock movements.

In the context of investing, the term portfolio refers to a combination of different types of assets, where funds are allocated specifically to each asset in that portfolio. The formation of an investment portfolio has varied objectives, depending on the individual preferences of each investor. The majority of investors expect an optimal portfolio. An optimal portfolio is a combination of investments that achieve the desired rate of return with minimal risk, or conversely, minimize risk for a certain rate of return.

2. Materials and Methods

2.1. Nonlinear Programming

Nonlinear programming is a method in operations research used to solve optimization problems by utilizing nonlinear equations and inequalities. The goal is to achieve optimal results taking into account the limitation of resources at a certain value. An optimization problem is classified as nonlinear if its goal and constraint functions have a nonlinear form, either one or both. Some of the factors that cause nonlinearity in the function of goals can stem from conditions such as price elasticity in large firms, where the amount of product that can be sold is inversely proportional to its price. In other words, the less product produced, the higher the price

For example, if each product of n its type of product has a similar profit function, defined $P_j(x_j)$ for the production and unit sales x_j of the product j where $(j = 1,2, \dots, n)$ the complete purpose function is

$$f(x) = \sum_{j=1}^{n} P_j(x_j) \tag{1}$$

2.2. Piecewise Linear Function Approach

Separable programming is an approach to deal with problems in nonlinear programming by converting nonlinear forms into linear forms involving only one variable. This method is related to handling nonlinear functions that are broken down into functions involving only one variable.

Definition 1: A function $f(x)$ can be said to be separate t if it can be expressed in addition

$$f(x) = f(x_1, x_2, \dots, x_n) = f(x_1) + f(x_2) + \cdots + f(x_n) = \sum_{j=1}^{n} f_j(x_j) \tag{2}$$

The above equation can be transformed into the form of an optimization problem

$$\text{Max/Min. } Z = \textstyle\sum_{j=1}^{n} f_j(x_j) \tag{3}$$

$$\text{s.t. } \textstyle\sum_{j=1}^{n} g_j(x_j)(\leq, =, \geq)\ b_i, i = 1,2, \dots, m \tag{4}$$

$$x_j \geq 0, (j = 1,2, \dots, n) \tag{5}$$

where f_j is a goal functio, g_j is a constraint function, b_i is a right field constant in the constraint function and x_j is an independent variable.

A piecewise linear function, also called a piecewise linear function, is a type of mathematical function consisting of straight line segments or different parts. The use of this function is generally applied in nonlinear problem solving, allowing a simpler and linear approach to situations that may be nonlinear in nature.

In dealing with nonlinear problems, the method of approximation using linear piecewise functions involves dividing the domain of nonlinear functions into certain intervals. At each interval, this approach replaces the nonlinear properties of the function with a linear function evaluated at that interval. By selecting smaller intervals and applying numerical methods, this approach can be applied to more complex nonlinear problems, where the corresponding straight-line equation is determined for each interval.

Definition of 2 Line Segments (Winston, 2004). Given $\bar{y}_1, \bar{y}_2 \in \mathbb{R}$. $L = \{\bar{y} | \bar{y} = \lambda\bar{y}_1 + (1-\lambda)\bar{y}_2, 0 \leq \lambda \leq 1\}$ called connecting line segments $\bar{y}_1$ and $\bar{y}_2$.

2.3. Portfolio

A portfolio refers to a combination of various securities that investors invest in and manage, both individuals and institutions. Such securities involve stocks, securities, bonds, certificates, and others. The definition of a portfolio includes the allocation of funds to a number of assets with a view to achieving future profits. An efficient portfolio refers to a combination of assets that maximize the expected rate of return with a certain risk or, conversely, offer the lowest risk with a certain rate of return.

Return

Return is the result obtained from an investment, and there is a positive relationship between return and risk in the context of investment known as high risk - high return. This means that the greater the risk taken, the greater the expected return. This concept reflects the need for additional returns in return for increased risk that must be borne by investors. Return can be in the form of realized return, which is the actual result that has occurred, or expected return, which is an estimate of expected return in the future. In general the total return between periods $t-1$ until t adalah sebagai berikut:

$$R_{it} = \frac{P_{it} - P_{i(t-1)}}{P_{i(t-1)}} \tag{6}$$

where R_{it} represents the return of shares to-i in the period t, P_{it} represents the closing price of the shares to-i pada periode ke-t and $P_{i(t-1)}$ represents the closing price of the shares to-i in the period to-$(t-1)$.

Expected Return

Expected return is the return that is expected to be obtained by investors in the future and has not yet occurred. Compared to historical returns, expected returns are important returns because they can be used as investment decision making. Expected return is simply a weighted average of various returns. The expected return of individual stocks can use the following formula:

$$E(R_i) = \frac{\sum_{t=1}^{n} R_{it}}{N} \tag{7}$$

where $E(R_i)$ is an expected return of shares to-i, R_{it} represents the return of shares to-i in the period t and N merupakan banyaknya return yang terjadi pada periode observasi.

Portfolio expected returns are the weighted average of the expected returns of each single stock in the notated portfolio $E(R_p)$ with the following equation:

$$E(R_p) = \sum_{i=1}^{n} x_i E(R_i) \tag{8}$$

where $E(R_p)$ is the expected return of the portfolio, $E(R_i)$ is an expected return of shares to-i, x_i is the proportion of funds invested in shares i and n represents the sum of a single share.

Risk

Risk is defined as the amount of deviation between the expected rate of return and the realized return. Financial risk is defined as the uncertainty of the future return of an investment, or that an investment gets a return that is smaller than the expected return and sometimes results in a loss, that is, a negative return. Types of risk can be grouped into two, namely systematic risk and unsystematic risk. One risk gauge is standard deviation or variance which is the square of the standard deviation. Individual stock risk

$$\sigma_i^2 = \frac{\sum_{t=1}^{n}(R_{it} - E(R_i))^2}{N} \tag{9}$$

where σ_i^2 is a stock risk to-i, R_{it} represents the return of shares to-i at period t, $E(R_i)$ is an expected return of shares to-i and N is the number of returns that occur in the observation period.

Risk can be interpreted as the degree of unexpected losses that depend on the composition of the portfolio. One risk gauge metric is standard deviation or variance, which is the square of the standard deviation. Measurement of portfolio risk can be done by calculating the variance of the return value of individual stocks in it. The number of stocks in a portfolio also has an impact on the value of risk variance. To form a portfolio, a minimum of two securities are needed, where the risk of both can be calculated through the variance of the values of those securities in the portfolio. The variance formula for a portfolio can be explained as follows.

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{10}$$

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j cov(R_i R_j), i \neq j \tag{11}$$

where σ_p^2 is portfolio risk, σ_i^2 is an individual stock risk, x_i is the proportion of funds invested in shares i and x_j is a fund invested in stocks j. With Formula $cov(R_i R_j)$ is

$$cov(R_i R_j) = \frac{\sum_{t=1}^{n}(R_{it} - E(R_i))(R_{jt} - E(R_j)}{N} \tag{12}$$

where $cov(R_i R_j)$ is a covariance of return between stocks i with shares j, R_{it} represents the return of shares to-i at period t, $E(R_i)$ is an expected return of shares to-i , R_{jt} represents the return of shares to-i at period t and $E(R_j)$ is an expected return of shares to-j.

3. Results and Discussion

The object of research in this case study is the weekly closing price data of Bank Jago Tbk. (ARTO), Barito Pacific Tbk. (BRPT) and Go To Gojek Tokopedia Tbk. (GOTO) taken from yahoofinance.com.

3.1. Return, Expected Return ARTO, BRPT dan GOTO

To find the return on portfolio stocks use the equation. The results of the calculation of return and expected return from stocks are as given in Table 1.

Table 1: Return and Expected Return of Stock ARTO, BRPT, GOTO

Date	Return		
	ARTO	BRPT	GOTO
12/5/2022			
12/12/2022	0.150141643	0	0.032258065
12/19/2022	-0.078817734	0.010689271	-0.104166667
12/26/2022	-0.005347594	-0.032051282	0.058139535
1/2/2023	-0.177419355	0.086092715	0.043956044
1/9/2023	-0.003267974	0.024390244	0.105263158
1/16/2023	0.052459016	-0.005952381	0.085714286
1/23/2023	0.037383178	0.005988024	0.026315789
1/30/2023	0.093093093	-0.011904762	0.05982906
2/6/2023	-0.156593407	0.030120482	-0.153225806
2/13/2023	-0.029315961	0.046783626	0.19047619
2/20/2023	-0.151006711	0.044692737	-0.04
2/27/2023	0.086956522	-0.122994652	0.033333333
3/6/2023	-0.047272727	-0.048780488	0.008064516
3/13/2023	-0.145038168	-0.012820513	-0.072
3/20/2023	0.0625	0.064935065	-0.00862069
3/27/2023	0.016806723	0.006097561	-0.052173913
4/3/2023	-0.037190083	-0.060606061	-0.073394495
4/10/2023	-0.055793991	0.038709677	-0.089108911
4/17/2023	-0.05	-0.00621118	0.032608696
4/24/2023	-0.009569378	0.04375	0.094736842
5/1/2023	0.053140097	-0.017964072	0.019230769
5/8/2023	0.142201835	-0.018292683	0.094339623
5/15/2023	-0.032128514	-0.01242236	-0.00862069
5/22/2023	-0.029045643	-0.025157233	-0.095652174
5/29/2023	0.017094017	-0.019354839	0.413461538

6/5/2023	0.340336134	-0.065789474	-0.136054422
6/12/2023	-0.084639498	0.084507042	-0.078740157
6/19/2023	0.051369863	-0.038961039	-0.034188034
6/26/2023	0.035830619	0.006756757	-0.026548673
7/3/2023	0.01572327	0.020134228	-0.018181818
7/10/2023	-0.009287926	0.013157895	0.046296296
7/17/2023	-0.05	0.006493506	0
7/24/2023	-0.009868421	-0.012903226	-0.017699115
7/31/2023	-0.119601329	0.019607843	-0.027027027
8/7/2023	-0.018867925	0.096153846	-0.157407407
8/14/2023	-0.003846154	0.046783626	0.010989011
8/21/2023	-0.065637066	0.150837989	-0.086956522
8/28/2023	-0.049586777	0.165048544	0.130952381
9/4/2023	0	-0.05	-0.021052632
9/11/2023	-0.02173913	0.245614035	-0.010752688
9/18/2023	-0.008888889	0.045774648	-0.054347826
9/25/2023	-0.094170404	-0.124579125	-0.022988506
10/2/2023	-0.012376238	0.042307692	-0.011764706
10/9/2023	-0.187969925	-0.15498155	-0.202380952
10/16/2023	0.055555556	-0.109170306	-0.104477612
10/23/2023	-0.058479532	-0.029411765	-0.066666667
10/30/2023	0.118012422	0.050505051	0.25
11/6/2023	0.144444444	0.129807692	0.085714286
11/13/2023	0.111650485	-0.008510638	0.105263158
11/20/2023	0.349344978	-0.154506438	0.119047619
11/27/2023	0.129449838	0.030456853	0.159574468
12/4/2023	-0.140401146	0.57635468	-0.128440367
12/8/2023	0.023333333	0.08125	0.052631579

Individual stock risk is obtained using equations and covariances of return between ARTO, BRPT, GOTO stocks can be found using equations (11) and (12), so that it is obtained as shown in Table 2.

Table 2: Risk Individual and Covarian Stock between ARTO, BRPT, GOTO

Risk of Individual Stock			**Covarian**		
ARTO	BRPT	GOTO	ARTO, BRPT	ARTO,GOTO	BRPT,GOTO
0.00053	0.00184	0.00060	-0.00315	0.00410	-0.00056

3.2. Optimal portfolio nonlinear model for stock investment ARTO, BRPT, GOTO

In the Markowitz Mean Variance model, there are two types of models in determining the proportion of funds, namely minimizing risk by setting expected returns first and maximizing expected returns by maintaining risk at a certain level. The formation of a nonlinear model in the optimal portfolio that combines two objective functions, namely maximizing expected return and minimizing risk. Defined decision variables $x_i, i = 1,2, \dots, n$ states the proportion of funds invested in stocks i. $R(x)$ as the expected return of the portfolio which is the total of the expected return of each stock. Soas the total portfolio risk of the risk of each stock, so that it obtains: the equation can be written as follows:

$$R(x) = \sum_{i=1}^{n} x_i E(R_i) \tag{13}$$

Define $P(x)$ as the total portfolio risk of the risk of each stock, so that it obtains:

$$P(x) = \sum_{i=1}^{n} x_i^2\, \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{14}$$

A nonlinear programming model in a portfolio that maximizes expected return with a certain level of risk, where the parameter α by being at intervals (0,1] is a non-negative constant that measures the level of investor desire for the relationship between expected return and risk. A nonlinear programming model for portfolios aimed at maximizing expected return with a certain level of risk can be formulated as follows (Frederick S, 2001)

$$\text{Max. } f(x) = R(x) - \alpha P(x) \tag{15}$$

$$\text{s.t. } \textstyle\sum_{j=1}^{n} x_j \le B \tag{16}$$

$$x_j \ge 0, (j = 1,2, \dots, n) \tag{17}$$

Where B is the amount of funds to be invested.

Suppose an investor will invest his funds in the amount of 90,000,000.00 in three companies, namely Bank Jago, Barito Pasicif and Go To Gojek Tokopedia. These investors are investors with *the risk averter* category or investors who are less willing to take risks. Assume α expressed as the level of risk to the investor by α is at intervals $(0,1]$. Choose $\alpha = 1$ based on assumptions and suppose variables x_1 the large proportion of funds to be invested in ARTO, x_2 the large proportion of funds to be invested in BRPT dan x_3 the large proportion of funds to be invested in GOTO.

$$f(x) = P(x) = \left(\sum_{i=1}^{n} x_i E(R_i)\right) - \alpha\left(\sum_{i=1}^{n} x_i^2\, \sigma_i^2 + 2x_i x_j cov\left(R_i R_j\right)\right) \tag{18}$$

Based on the data obtained the equation can be written in the form of:

$$\begin{aligned} f(x) = {} & (0.00271x_1 + 0.02020x_2 + 0.00671x_3) - \alpha(0.000531955x_1^2 + 0.001838407x_2^2 + 0.000603355x_3^2 \\ & + 2x_1x_2(-0.00315) + 2x_1x_3(0.00410) + 2x_2x_3(-0.00056) \end{aligned}$$

By using the triangular row of pascals obtained

$$\begin{aligned} f(x) = {} & (0.00271x_1 + 0.02020x_2 + 0.00671x_3) - \alpha(0.000531955x_1^2 + 0.001838407x_2^2 + 0.000603355x_3^2 \\ & - 0.00315(x_1 + x_2)^2 + 0.00315x_1^2 + 0.00315x_2^2 + 0.00410(x_1 + x_3)^2 - 0.00410x_1^2 \\ & - 0.00410x_3^2 - 0.00056(x_2 + x_3)^2 + 0.00056x_2^2 + 0.00056x_3^2) \end{aligned}$$

$$\begin{aligned} f(x) = {} & (0.00271x_1 + 0.02020x_2 + 0.00671x_3) - \alpha(-0.000418045x_1^2 + 0.005548407x_2^2 - 0.002936645x_3^2 \\ & - 0.00315(x_1 + x_2)^2 + 0.00410(x_1 + x_3)^2 - 0.00056(x_2 + x_3)^2) \end{aligned}$$

Subs. $\alpha = 1$, Retrieved

$$\begin{aligned} f(x) = {} & 0.00271x_1 + 0.02020x_2 + 0.00671x_3 + 0.000418045x_1^2 - 0.005548407x_2^2 + 0.002936645x_3^2 \\ & + 0.00315(x_1 + x_2)^2 - 0.00410(x_1 + x_3)^2 + 0.00056(x_2 + x_3)^2 \end{aligned}$$

Assume $x_1 + x_2 = x_4$, $x_1 + x_3 = x_5$ dan $x_2 + x_3 = x_6$, so that the equation becomes

$$\begin{aligned} f(x) = {} & 0.00271x_1 + 0.02020x_2 + 0.00671x_3 + 0.000418045x_1^2 - 0.005548407x_2^2 + 0.002936645x_3^2 \\ & + 0.00315{x_4}^2 - 0.00410{x_5}^2 + 0.00056{x_6}^2 \end{aligned}$$

Acquired purpose function

Max. $f(x) = 0.00271x_1 + 0.02020x_2 + 0.00671x_3 + 0.000418045x_1^2 - 0.005548407x_2^2 + 0.002936645x_3^2 + 0.00315{x_4}^2 - 0.00410{x_5}^2 + 0.00056{x_6}^2$

With constraints $x_1 + x_2 + x_3 = 9$,

$$\begin{aligned} & x_1 + x_2 - x_4 = 0 \\ & x_1 + x_3 - x_5 = 0 \\ & x_2 + x_3 - x_6 = 0 \\ & x_1, x_2, x_3, x_4, x_5, x_6 \ge 0 \end{aligned}$$

3.3. Solve problems using separable functions with a piecewise linear approach

The optimization problems that have been formed are written in the form of the following separable function

$$f_1(x_1) = 0.00271x_1 + 0.000418045x_1^2$$

$$f_2(x_2) = 0.02020x_2 - 0.005548407x_2^2$$
$$f_3(x_3) = 0.00671x_3 + 0.002936645x_3^2$$
$$f_4(x_4) = 0.00315{x_4}^2$$
$$f_5(x_5) = -0.00410{x_5}^2$$
$$f_6(x_6) = 0.00056{x_6}^2$$

With constraints

$$g_{11}(x_1) = x_1,\ g_{12}(x_2) = x_2,,\ g_{13}(x_3) = x_3$$
$$g_{21}(x_1) = x_1,\ g_{22}(x_2) = x_2, g_{24}(x_3) = -x_4$$
$$g_{31}(x_1) = x_1,\ g_{33}(x_3) = x_3, g_{35}(x_5) = -x_5$$
$$g_{42}(x_2) = x_2,\ g_{43}(x_3) = x_3, g_{46}(x_6) = -x_6$$

Suppose k represents the number of partition points or the number of iterations. Select as many iterations as 9 with intervals [0;9]. So the Table 3 of values is discrete for the destination function:

Table 3 Discrete values for destination functions

f1k	f2k	f3k	f4k	f5k	f6k
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00313	0.01465	0.00965	0.00315	-0.00410	0.00056
0.00709	0.01821	0.02517	0.01260	-0.01640	0.00224
0.01189	0.01066	0.04656	0.02835	-0.03690	0.00504
0.01753	-0.00797	0.07383	0.05040	-0.06560	0.00896
0.02400	-0.03771	0.10697	0.07875	-0.10250	0.01400
0.03131	-0.07854	0.14598	0.11340	-0.14760	0.02016
0.03945	-0.13047	0.19087	0.15435	-0.20090	0.02744
0.04843	-0.19350	0.24163	0.20160	-0.26240	0.03584
0.05825	-0.26762	0.29826	0.25515	-0.33210	0.04536

With a discrete table of values for the constraint function as shown in Table 4.

Table 4 Discrete values for constraint functions

g11k	g12k	g13k	g21k	g22k	g24k	g31k	g33k	g35k	g42k	g43k	g46k
0	0	**0**	**0**	0	**0**	**0**	**0**	0	0	**0**	0
1	1	1	1	1	-1	1	1	-1	1	1	-1
2	2	2	2	2	-2	2	2	-2	2	2	-2
3	3	3	3	3	-3	3	3	-3	3	3	-3
4	4	4	4	4	-4	4	4	**-4**	4	4	-4
5	**5**	5	5	**5**	-5	5	5	**-5**	**5**	5	-5
6	6	6	6	6	-6	6	6	-6	6	6	-6
7	7	7	7	7	-7	7	7	-7	7	7	-7
8	8	8	8	8	-8	8	8	-8	8	8	-8
9	9	**9**	9	9	**-9**	9	**9**	-9	9	**9**	**-9**

Based on the Table 4, the purpose functions are:

$$\begin{aligned}\hat{f}(x_{ij}) = {} & 0\lambda_{11} + 0\lambda_{12} + 0.01\lambda_{13} + 0.01\lambda_{14} + 0.02\lambda_{15} + 0.02\lambda_{16} + 0.03\lambda_{17} + 0.04\lambda_{18} + 0.05\lambda_{19} + 0.06\lambda_{110} \\ & + 0\lambda_{21} + 0\lambda_{22} + 0.02\lambda_{23} + 0.01\lambda_{24} - 0.01\lambda_{25} - 0.04\lambda_{26} - 0.08\lambda_{27} - 0.13\lambda_{28} - 0.19\lambda_{29} \\ & - 0.27\lambda_{210} + 0\lambda_{31} + 0.01\lambda_{32} + 0.03\lambda_{33} + 0.05\lambda_{34} + 0.07\lambda_{35} + 0.11\lambda_{36} + 0.15\lambda_{37} + 0.19\lambda_{38} \\ & + 0.24\lambda_{39} + 0.30\lambda_{310} + 0\lambda_{41} + 0\lambda_{42} + 0.01\lambda_{43} + 0.03\lambda_{44} + 0.05\lambda_{45} + 0.08\lambda_{46} + 0.11\lambda_{47} \\ & + 0.15\lambda_{48} + 0.20\lambda_{49} + 0.26\lambda_{410} + 0\lambda_{51} + 0\lambda_{52} - 0.02\lambda_{53} - 0.04\lambda_{54} - 0.07\lambda_{55} - 0.10\lambda_{56} \\ & - 0.15\lambda_{57} - 0.20\lambda_{58} - 0.26\lambda_{59} - 0.33\lambda_{510} + 0\lambda_{61} + 0\lambda_{62} + 0\lambda_{63} + 0.01\lambda_{64} + 0.01\lambda_{65} + 0.01\lambda_{66} \\ & + 0.02\lambda_{67} + 0.03\lambda_{68} + 0.04\lambda_{69} + 0.05\lambda_{610}\end{aligned}$$

With constraints

$$\widehat{g_{1j}}(x_{ij}) = 0\lambda_{11} + 1\lambda_{12} + 2\lambda_{13} + 3\lambda_{14} + 4\lambda_{15} + 5\lambda_{16} + 6\lambda_{17} + 7\lambda_{18} + 8\lambda_{19} + 9\lambda_{110} + 0\lambda_{21} + 1\lambda_{22} + 2\lambda_{23} + 3\lambda_{24} + 4\lambda_{25} + 5\lambda_{26} + 6\lambda_{27} + 7\lambda_{28} + 8\lambda_{29} + 9\lambda_{210} + 0\lambda_{31} + 1\lambda_{32} + 2\lambda_{33} + 3\lambda_{34} + 4\lambda_{35} + 5\lambda_{36} + 6\lambda_{37} + 7\lambda_{38} + 8\lambda_{39} + 9\lambda_{310}$$

$$\widehat{g_{2j}}(x_{ij}) = 0\lambda_{11} + 1\lambda_{12} + 2\lambda_{13} + 3\lambda_{14} + 4\lambda_{15} + 5\lambda_{16} + 6\lambda_{17} + 7\lambda_{18} + 8\lambda_{19} + 9\lambda_{110} + 0\lambda_{21} + 1\lambda_{22} + 2\lambda_{23} + 3\lambda_{24} + 4\lambda_{25} + 5\lambda_{26} + 6\lambda_{27} + 7\lambda_{28} + 8\lambda_{29} + 9\lambda_{210} - 0\lambda_{41} - 1\lambda_{42} - 2\lambda_{43} - 3\lambda_{44} - 4\lambda_{45} - 5\lambda_{46} - 6\lambda_{47} - 7\lambda_{48} - 8\lambda_{49} - 9\lambda_{410}$$

$$\widehat{g_{3j}}(x_{ij}) = 0\lambda_{11} + 1\lambda_{12} + 2\lambda_{13} + 3\lambda_{14} + 4\lambda_{15} + 5\lambda_{16} + 6\lambda_{17} + 7\lambda_{18} + 8\lambda_{19} + 9\lambda_{110} + 0\lambda_{31} + 1\lambda_{32} + 2\lambda_{33} + 3\lambda_{34} + 4\lambda_{35} + 5\lambda_{36} + 6\lambda_{37} + 7\lambda_{38} + 8\lambda_{39} + 9\lambda_{310} - 0\lambda_{51} - 1\lambda_{52} - 2\lambda_{53} - 3\lambda_{54} - 4\lambda_{55} - 5\lambda_{56} - 6\lambda_{57} - 7\lambda_{58} - 8\lambda_{59} - 9\lambda_{510}$$

$$\widehat{g_{4j}}(x_{ij}) = 0\lambda_{21} + 1\lambda_{22} + 2\lambda_{23} + 3\lambda_{24} + 4\lambda_{25} + 5\lambda_{26} + 6\lambda_{27} + 7\lambda_{28} + 8\lambda_{29} + 9\lambda_{210} + 0\lambda_{31} + 1\lambda_{32} + 2\lambda_{33} + 3\lambda_{34} + 4\lambda_{35} + 5\lambda_{36} + 6\lambda_{37} + 7\lambda_{38} + 8\lambda_{39} + 9\lambda_{310} - 0\lambda_{61} - 1\lambda_{62} - 2\lambda_{63} - 3\lambda_{64} - 4\lambda_{65} - 5\lambda_{66} - 6\lambda_{67} - 7\lambda_{68} - 8\lambda_{69} - 9\lambda_{610}$$

The above problem is solved using the Simplex table and obtained:
$\lambda_{11} = 1, \lambda_{26} = 1, \lambda_{31} = 0.55556, \lambda_{310} = 0.44444, \lambda_{41} = 0.44444, \lambda_{410} = 0{,}55556, \lambda_{55} = 1, \lambda_{55} = 0, \lambda_{610} = 0, \lambda_{610} = 1$ and others of value 0
So that the optimal value of the goal function is obtained, namely:

$$\hat{f}(x_{ij}) = 0.21636$$

and the value of x_1, x_2, x_3, x_5, x_6

$$x_1 = 0(1) + 1\lambda_{12} + 2\lambda_{13} + 3\lambda_{14} + 4\lambda_{15} + 5\lambda_{16} + 6\lambda_{17} + 7\lambda_{18} + 8\lambda_{19} + 9\lambda_{110} = 0$$
$$x_2 = 0\lambda_{21} + 1\lambda_{22} + 2\lambda_{23} + 3\lambda_{24} + 4\lambda_{25} + 5(1) + 6\lambda_{27} + 7\lambda_{28} + 8\lambda_{29} + 9\lambda_{210} = 5$$
$$x_3 = 0(0.55556) + 1\lambda_{32} + 2\lambda_{33} + 3\lambda_{34} + 4\lambda_{35} + 5\lambda_{36} + 6\lambda_{37} + 7\lambda_{38} + 8\lambda_{39} + 9(0.44444) = 3.9996$$
$$x_4 = 0(0.44444) + 1\lambda_{42} + 2\lambda_{43} + 3\lambda_{44} + 4\lambda_{45} + 5\lambda_{46} + 6\lambda_{47} + 7\lambda_{48} + 8\lambda_{49} + 9(0.55556) = 5.0004$$
$$x_5 = 0\lambda_{51} + 1\lambda_{52} + 2\lambda_{53} + 3\lambda_{54} + 4(1) + 5(0) + 6\lambda_{57} + 7\lambda_{58} + 8\lambda_{59} + 9\lambda_{510} = 4$$
$$x_6 = 0(0) + 1\lambda_{62} + 2\lambda_{63} + 3\lambda_{64} + 4\lambda_{15} + 5\lambda_{66} + 6\lambda_{67} + 7\lambda_{68} + 8\lambda_{69} + 9(1) = 9$$

Proportion of each stock

Bank Jago Tbk. (ARTO): $x_1 = 0 \times 10.000.000 = 0$
Barito Pacific Tbk. (BRPT): $x_2 = 5 \times 10.000.000 = 50{,}000{,}000$
Go To Gojek Tokopedia Tbk. (GOTO): $x_3 = 3.9996 \times 10.000.000 = 39{,}999{,}600$

If a grade is provided α that is 0.01; 1 Then the value of the Destination function is in Table 5 below:

$$f(x) = (0.00271x_1 + 0.02020x_2 + 0.00671x_3) - \alpha(-0.000418045x_1^2 + 0.005548407x_2^2 - 0.002936645x_3^2 - 0.00315(x_1 + x_2)^2 + 0.00410(x_1 + x_3)^2 - 0.00056(x_2 + x_3)^2)$$

Table 5 Value the Goal Function with Different Proportions of Funds

α	Bank Jago Tbk. (ARTO): x_1	Barito Pacific Tbk. (BRPT): x_2	Go To Gojek Tokopedia Tbk. (GOTO): x_3	$f(x)$
0.01	0	90,000,000	0	1,803,109
1	0	50,000,000	39,999,600	1,275,052

From the perspective of an investor who tends to dare to take risks with a level of α as big as 0.01, The expected profit from the investment of all three shares in the calculation period reaches IDR 1,803,109. On the other hand, for investors who are more cautious and have a Rate α as big as 1, The anticipated profit in the same investment is approximately IDR 1,275,052.

4. Conclussion

The piecewise linear approach in nonlinear programming proves its effectiveness by breaking nonlinear functions into straight-line segments between discrete points. Although it can overcome the complexity of calculations, its accuracy depends on the extent to which linear approximations can represent the true characteristics of nonlinear functions. In financial contexts, such as stock investing, piecewise linear can be used as a solution to solve optimization problems, assisting investors in managing their investment portfolios by achieving desired returns and minimal risk. It is important for investors to understand that engaging in stock investing also carries risk, and careful monitoring of stock movements is key to managing that risk. By forming an optimal portfolio, investors can achieve a balance between investment returns and acceptable levels of risk.

In this study, the stock case that was resolved was the LQ45 stock portfolio, namely ARTO, BRPT, GOTO. The calculation results show an investor who tends to dare to take risks with a level of α as big as 0.01, The expected profit from the investment of all three shares in the calculation period reaches IDR 1,803,109. On the other hand, for investors who are more cautious and have a Rate α as big as 1, The anticipated profit in the same investment is approximately IDR 1,275,052.

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