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Multiobjective Optimization of Stock Portofolio

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Abstract

Diversification is a method used to reduce risks by allocating several financial, industrial, and other instruments. Investors might need to use this method to allocate their companies' funding as efficient as they should be. Mean variance portfolio is a diversification theory designated for investors who are aware of potential risks. On the other hand, multi-objective portfolio optimization is another theory that suits for investors who are more unaware, or choose to neglect potential business risks. Multi-objective optimization can boost source of income and minimize the risks while utilizing k weighting coefficient as risk aversion index. This research aims to form an optimal portfolio from each perspective of selected investors using multi-objective optimization, as well as to analyze the benefits and risks that the investors will have. Samples used in this research are sharia stocks actively involved in Jakarta Islamic Index (JII) and non-sharia stocks which are actively involved in LQ-45 from January 2013 to January 2018.

Keywords: Diversification, multi-objective optimization, optimal portfolio

1. Introduction

Statistics is a part of mathematics that specifically discusses ways of collecting, analyzing and interpreting data. In other words, the term statistics is used to denote a body of knowledge about methods of sampling (data collection), as well as analysis and interpretation of data (Lock et al., 2020). With statistics, you can explain the relationships between variables, make better decisions, overcome changes that occur and make plans and forecasts. Statistics is widely applied in various sciences, ranging from social sciences to the business sector. One of the uses of statistics in the economic field is determining the risk and return of a portfolio in determining investment in the capital market.

The capital market is a means of funding for companies and governments, and as a means of investment activities for fund owners (Black & Gilson, 1998). Thus, the capital market facilitates various facilities and infrastructure for buying and selling activities and other related activities. The capital market has a big role in a country's economy because the capital market carries out two functions at once, namely the economic function and the financial function (Stulz, 2001). In conditions of a financial system where the level of volatility is considered high, the sharia financial system, especially the sharia capital market, offers an attractive alternative compared to the conventional industry which still relies on usury as its basic foundation.

According Dixit & Pindyck (1994), investment can be interpreted as a commitment to invest a certain amount of funds now with the aim of obtaining profits in the future. Investment can relate to investing a certain amount of funds in real stocks such as: land, gold, houses and other real stocks or in financial stocks such as: deposits, stocks, bonds and other securities (Siegel, 2021). Securities traded on the Indonesian capital market in the form of equity are stocks, both ordinary stocks and preferred stocks as well as evidence of rights and warrants. Of the four equity securities, ordinary stocks are the most important and best known securities by the Indonesian people. Therefore, the term equity market is often understood as the stock market and the term stocks are often meant to mean ordinary stocks. Once issued, equity securities can be traded by investors on the stock exchange. Currently in Indonesia there is one stock exchange, namely the Indonesian Stock Exchange (BEI).

Investing in risky stocks such as stocks will not only generate profits (returns) but also have to face losses (risk). The rate of return can be measured from expected return while risk can be measured from variance or standard deviation. Therefore, diversification is needed to reduce risk. Professor Harry Markowitz is the originator of portfolio diversification theory, which became known as Markowitz's diversification theory. Markowitz's efficient portfolio concept is also called the Mean Variance Efficient Portfolio (MVEP). Kandel (1984) defines the Mean Variance Efficient Portfolio (MVEP) as a portfolio that has the minimum variance among all possible portfolios that can be formed. Rachev et al. (2008) said that the optimal portfolio depends on investor preferences. Weston (1974) divided types of investors into three groups based on their preferences, namely groups who like to face risks (risk seekers), anti-risk groups (risk averse), and groups who are indifferent to risk (risk indifference).

The portfolio concept coined by Markowitz is portfolio optimization aimed at standard investors because it only refers to one explanation of portfolio returns (Balqis et al., 2021). Duan (2007) uses a multiobjective approach to form an optimal portfolio by maximizing returns and minimizing risk simultaneously. Multiobjective optimization offers several investment alternatives depending on the investor's preferences. Based on this background, a problem can be formulated, namely how to find the optimal stock portfolio using the Multiobjective Optimization Method for various types of investors.

2. Materials and Methods

2.1. Materials

In this research, the data used is a list of sharia and non-sharia stocks listed on the Indonesia Stock Exchange (BEI) which was downloaded from the site www.finance.yahoo.com. The data source that will be used as the object of research is the monthly close price value of Jakarta Islamic Index (JII) and LQ-45 stocks for the period January 2013-January 2018. The stocks selected for the sharia portfolio consist of 17 stocks that were consistent during the period January 2013-January 2018 was included in the Jakarta Islamic Index (JII). Meanwhile, the non-shariah portfolio is formed from 10 stocks that consistently during the period January 2013-January 2018 were included in LQ-45 and were never included in the Jakarta Islamic Index (JII).The data analysis method used is a portfolio model with a multi-objective optimization approach. In calculating the multi-objective optimization portfolio model, the author used the software R i386 3.3.2, IBM SPSS Statistics 17 and Microsoft Excel 2010.

2.2. Methods

The steps for implementing multiobjective optimization are as follows:

a. Input P_{it}, where P_{it} closing price of stock i in period t, $i = 1,2,\dots,m$ and $t = 1,2,\dots,n$.

b. Calculate the return value of each stock using the formula

$$R_{it} = \frac{P_{it} - P_{it-1}}{P_{it-1}}. \tag{1}$$

with
R_{it} : return of stock i in period $t, t = 1,2,\dots,n$
P_{it} : price of stock i in period $t, t = 1,2,\dots,n$
P_{it-1} : price of stock i in period $t-1, t = 1,2,\dots,n$

c. Conduct a normality test on stock returns using the Kolmogorov Smirnov normality test.
Based on probability values

- Hypothesis
  H_0 : data is normally distributed
  H_1: data is not normally distributed
- Test criteria
  If the significance value is $>$ 0.05 then H_0 is accepted
  If the significance value is $<$ 0.05 then $H_0$is rejected

d. If the data does not meet the normal test, then look for new data then repeat steps 2 and 3 until stock returns are normally distributed. If the data is normally distributed, then do the next step.

e. Calculate average return of stock

$$\bar{R}_i = \frac{\sum R_{it}}{n} \tag{2}$$

with
$\bar{R}_i$: average return of stock i, $i = 1,2,\dots,m$
R_{it} : return of stock i in period t and $t = 1,2,\dots,n$

f. Calculate standar deviation of stock

$$\sigma_i = \sqrt{\frac{\sum (R_{it} - \bar{R}_i)^2}{n-1}} \tag{3}$$

with
σ_i : standard deviation of stock i, $i = 1,2,\dots,m$
$\bar{R}_i$: average return of stock i, $i = 1,2,\dots,m$
R_{it} : return of stock i in period t, $t = 1,2,\dots,n$

g. Calculate ratio of stock

$$RS_i = \frac{\bar{R}_{it}}{\sigma_i} \tag{4}$$

with
RS_i: ratio of stock i, $i = 1,2,\dots,m$
$\bar{R}_{it}$: average return of stock $i, i = 1,2,\dots,m$
σ_i : standard deviation of stock $i, i = 1,2,\dots,m$

h. Calculate the variance-covariance matrix value.
i. Form an optimal stock portfolio by calculating the weight of each stock using multi-objective optimization with a combination of different k values based on the formula

$$\bar{x}^* = \frac{1}{2k}V^{-1}\bar{p} - \frac{V^{-1}}{2k}\left(\frac{a_2}{a_1} - \frac{2k}{a_1}\right)\bar{I} \tag{5}$$

with
$\bar{x}^*$: optimal weight of the multiobjective portfolio
V^{-1} : inverse variance covariance matrix
$\bar{p}$: column vector consisting of single stock returns
k : weighting coefficient
$\bar{I}$: column matrix with element 1

j. Calculate the return value of each optimal stock portfolio formed using the formula

$$E(R_p) = p^T x = \sum_{i=1}^{n} x_i p_i \tag{6}$$

with
x_i : portfolio weight of stock i
p_i : column vector consisting of stock return i

k. Determine the 95% confidence level value or α=0.05.
l. Calculate the Value at Risk (VaR) value of each optimal stock portfolio formed using the formula

$$VaR = \alpha\, \sigma_i\, x_i \tag{7}$$

with
α : level of confidence
σ_i : standard deviation of the ith stock
x_i : weight of the ith stock in the portfolio

m. Compare return values and Value at Risk (VaR) to determine the optimal stock portfolio that is suitable for investors according to their preferences

3. Results and Discussion

Based on 17 stocks given on Table 1, that are always consistently included in the Jakarta Islamic Index (JII) list, stocks were selected that had a positive mean return value and 14 stocks were obtained that had a positive mean return value. Based on 10 stocks that are always consistently included in the LQ45 list and never included in the Jakarta Islamic Index (JII) list, stocks were selected that had a positive mean return value and 9 stocks were obtained that had a positive mean return value.

Based on the results of the normality test using Kolmogorov-Smirnov, it appears that the significance value for the stocks of ICBP (Indofood CBP Sukses Makmur Tbk.), KLBF (Kalbe Farma Tbk.), LPPF (Matahari Department Store Tbk.), and UNVR (Unilever Indonesia Tbk.) $< \alpha = 0.05$ then H_0 is rejected. So it can be concluded that the stock data is not normally distributed. ICBP, KLBF, LPPF and UNVR stocks are not normally distributed because there is outlier data, so the outlier data is removed so that these stocks have a normal distribution.

Based on the results of the normality test using Kolmogorov-Smirnov, it appears that the significance value for BMRI (Bank Mandiri (Persero) Tbk.) stocks is $< \alpha = 0.05$, so H_0 is rejected. So it can be concluded that the stock data is not normally distributed. BMRI stocks are not normally distributed because there is outlier data, so the outlier data is removed so that the stocks have a normal distribution. Data that is not normally distributed does not meet the assumptions of parametric statistical tests.

Table 1: Selection of Jakarta Islamic Index (JII) Stock Portfolio

Code	Company Name	Sector
ADRO	Adaro Energy Tbk.	Mining
AKRA	AKR Corporindo Tbk.	Trade, Services and Investment
ASII	Astra International Tbk.	Various Industries
BSDE	Bumi Serpong Damai Tbk.	Property and Real Estate
INDF	Indofood Sukses Makmur Tbk.	Consumer Goods Industry
KLBF	Kalbe Farma Tbk.	Consumer Goods Industry
LPPF	Matahari Department Store Tbk.	Trade, Services and Investment
LSIP	PP London Sumatra Indonesia Tbk.	Agriculture
PTPP	PP (Persero) Tbk.	Property and Real Estate
SMRA	Summarecon Agung Tbk.	Property and Real Estate
TLKM	Telekomunikasi Indonesia (Persero) Tbk.	Infrastructure, Utilities, and Transportation
UNTR	United Tractors Tbk.	Trade, Services and Investment
UNVR	Unilever Indonesia Tbk.	Consumer Goods Industry

In a stock portfolio, different company sectors will minimize losses if one sector declines. To find out whether the 13 selected stocks in Table 2, are optimal or not, these stocks will be formed into an optimal portfolio using a multi-objective optimization approach.

Table 2: LQ-45 Stock Portfolio Selection

Code	Company Name	Sector
BBCA	Bank Central Asia Tbk.	Finance
BBNI	Bank Negara Indonesia (Persero) Tbk.	Finance
BBRI	Bank Rakyat Indonesia (Persero) Tbk.	Finance
BMRI	Bank Mandiri (Persero) Tbk.	Finance
GGRM	Gudang Garam Tbk.	Consumer Goods Industry
INTP	Indocement Tunggal Prakarsa Tbk.	Basic Industry and Chemistry
JSMR	Jasa Marga (Persero) Tbk.	Infrastructure, Utilities and Transportation
MNCN	Media Nusantara Citra Tbk.	Trade, Services and Investment
PTBA	Tambang Batubara Bukit Asam (Persero) Tbk.	Mining

In a stock portfolio, different company sectors will minimize losses if one sector declines. To find out whether the 9 selected stocks are optimal or not, these stocks will be formed into an optimal portfolio using a multi-objective optimization approach.

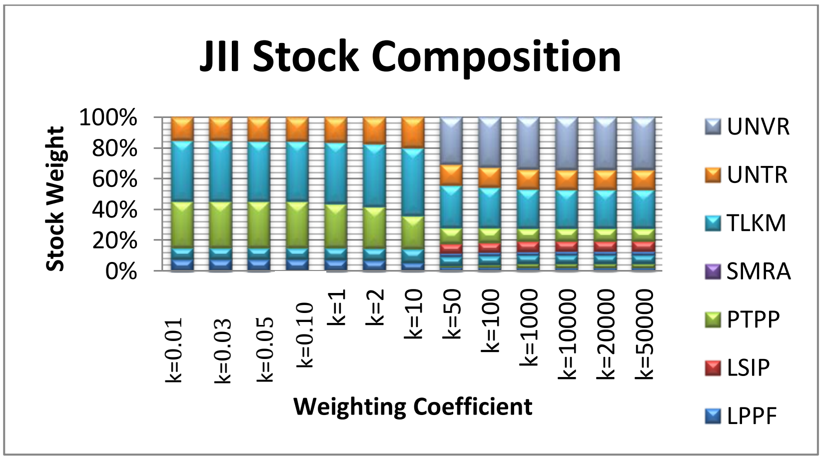

Figure 1: Composition of Jakarta Islamic Index (JII) Stock

Based on Figure 1, it appears that using a weighting coefficient $k = 0.01$ (k is close to zero) the stock that has the largest weight is Telekomunikasi Indonesia (Persero) Tbk.. (TLKM). Investing in stocks with a weighting coefficient $k = 0.01$ is suitable for investors with the risk seeker type. With a weighting coefficient of $k = 1$, the weight of Telekomunikasi Indonesia (Persero) Tbk.. (TLKM) stocks increases to 40.29% and the remainder is invested in Adaro Energy Tbk stocks. (ADRO) of 7.30%, Indofood Sukses Makmur Tbk. (INDF) of 7.87%, PP (Persero) Tbk. (PTPP) of 28.38%, and United Tractors Tbk. (UNTR) of 16.15%. By using a weighting coefficient of $k = 100$, the stock weight of Telekomunikasi Indonesia (Persero) Tbk.. (TLKM) is reduced from 40.29% to 26.68%. The remainder was invested in Adaro Energy Tbk. (ADRO) of 2.25%, Astra International Tbk. (ASII) of 1.93%, Indofood Sukses Makmur Tbk. (INDF) of 5.70%, Matahari Department Store Tbk. (LPPF) of 2.28%, PP London Sumatra Indonesia Tbk. (LSIP) of 6.34%, PP (Persero) Tbk. (PTPP) of 9.42%, United Tractors Tbk. (UNTR) of 13.12%, and Unilever Indonesia Tbk. (UNVR) of 32.27%. At a coefficient of $1 \leq k \leq 100$ there is a significant change in the stock weight, this weighting coefficient is suitable for use by risk indifference type investors, whereas at a weighting coefficient of $k > 100$ there is no significant change in the stock weight. A weighting coefficient that is close to an infinite number is suitable for use by risk averse investors.

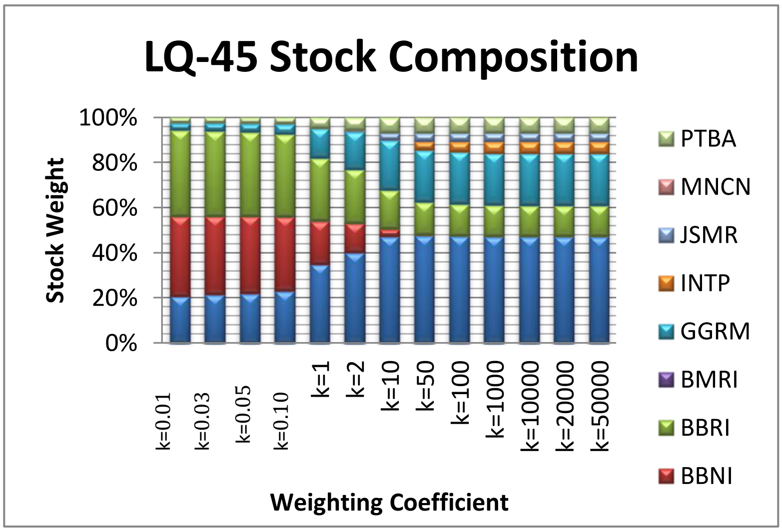

Figure 2: Composition of LQ-45 Stock

Based on Figure 2, it appears that using a weighting coefficient $k = 0.01$ (k is close to zero) the stock that has the largest weight is Bank Rakyat Indonesia Tbk. (BBRI). Investing in stocks with a weighting coefficient $k = 0.01$ is suitable for investors with the risk seeker type. With a weighting coefficient of $k = 1$, the stock weight of Bank Rakyat Indonesia Tbk. (BBRI) has been reduced to 27.90% and the remainder is invested in stocks of Bank Central Asia Tbk. (BBCA) of 34.98%, Bank Negara Indonesia (Persero) Tbk. (BBNI) of 19.28%, Gudang Garam Tbk. (GGRM) of 13.13%, and Bukit Asam Coal Mine (Persero) Tbk. (PTBA) of 4.71%. By using a weighting coefficient $k = 100$ to weight the stocks of Bank Rakyat Indonesia Tbk. (BBRI) further reduced from 27.90% to 14.23%. The remainder was invested in stocks of Bank Central Asia Tbk. (BBCA) of 47.55%, Gudang Garam Tbk. (GGRM) of 23.08%, Indocement Tunggal Prakarsa Tbk. (INTP) of 4.48%, Jasa Marga (Persero) Tbk. (JSMR) by 3.74%, and Bukit Asam Coal Mine (Persero) Tbk. (PTBA) of 6.91%. At a coefficient of $1 \leq k \leq 100$ there is a significant change in the stock weight, this weighting coefficient is suitable for use by risk indifference type investors, whereas at a weighting coefficient of $k > 100$ there is no significant change in the stock weight. A weighting coefficient that is close to an infinite number is suitable for use by risk averse investors.

In this case study, if an example is taken by investing capital of IDR 50,000,000.00, the expected profit in each portfolio with different weighting coefficients are as given in Table 3.

Table 3: Expected Return Value and Expected Profit for Each Jakarta Islamic Index (JII) Stock Portfolio

k	Expected Return	Profit
0.01	0.017922481	IDR 896,124.00
0.03	0.017914862	IDR 895,743.00
0.05	0.017907902	IDR 895,395.00
0.1	0.017890864	IDR 894,543.00
1	0.017626695	IDR 881,335.00
2	0.017394803	IDR 869,740.00
10	0.016497234	IDR 824,862.00
50	0.011783962	IDR 589,198.00
100	0.011373857	IDR 568,693.00
1000	0.010995057	IDR 549,753.00
10000	0.010958264	IDR 547,913.00
20000	0.010958922	IDR 547,946.00
50000	0.010955717	IDR 547,78.00

Table 3 shows the estimated profits that will be obtained by each investor according to their preferences. A small k value indicates that the investor wants large profits and a larger k value indicates that the investor does not expect large profits, as shown in the Table 4.

Table 4: Expected Return Value and Expected Profit for Each LQ45 Stock Portfolio

k	Expected Return	Profit
0.01	0.017009007	IDR 850,450.00
0.03	0.016960658	IDR 848,033.00
0.05	0.016917499	IDR 845,875.00
0.1	0.016807819	IDR 840,391.00
1	0.015718971	IDR 785,949.00
2	0.015165233	IDR 758,262.00
10	0.014159683	IDR 707,984.00
50	0.013475110	IDR 673,756.00
100	0.013368199	IDR 668,410.00
1000	0.013284962	IDR 664,248.00
10000	0.013275542	IDR 663,777.00
20000	0.013275923	IDR 663,796.00
50000	0.013275923	IDR 663,796.00

Table 4 shows the estimated profits that will be obtained by each investor according to their preferences. A small k value indicates that the investor wants large profits and a larger k value indicates that the investor does not expect large profits.

In this case study, if an example is taken by investing capital of IDR 50,000,000.00 and at a confidence level of 95%, then the Value at Risk (VaR) value for each portfolio with different weighting coefficients can be seen in the following Table 5.

Table 5: Value at Risk (VaR) for Each Jakarta Islamic Index (JII) Stock Portfolio

k	Value at Risk
0.01	IDR 212,515.00
0.03	IDR 212,428.00
0.05	IDR 212,359.00
0.1	IDR 212,181.00
1	IDR 209,400.00
2	IDR 206,970.00
10	IDR 197,544.00
50	IDR 176,248.00
100	IDR 174,235.00
1000	IDR 172,382.00
10000	IDR 172,233.00
20000	IDR 172,244.00
50000	IDR 172,226.00

Table 5 shows the estimated worst loss that each investor will incur according to their preferences. A small k value indicates that the investor likes risk and a larger k value indicates that the investor is increasingly risk averse, as shown in the Table 6.

Table 6: Value at Risk (VaR) for Each LQ45 Stock Portfolio

k	Value at Risk
0.01	IDR 194,798.00
0.03	IDR 194,410.00
0.05	IDR 194,054.00
0.1	IDR 193,139.00
1	IDR 184,266.00
2	IDR 180,553.00
10	IDR 175,922.00
50	IDR 176,163.00
100	IDR 176,361.00
1000	IDR 176,642.00
10000	IDR 176,653.00
20000	IDR 176,675.00
50000	IDR 176,675.00

Table 6 shows the estimated worst loss that each investor will experience according to their preferences. A small k value indicates that the investor likes risk and a larger k value indicates that the investor is increasingly risk averse.

4. Conclussion

After analyzing the data on the case study which is the object of research, the author can draw the following conclusions:

a. Portfolio formation using a multiobjective optimization approach considers the weighting coefficient (k value). By selecting a weighting coefficient that is close to zero, capital is allocated to stocks that have the highest rate of return, while by selecting a weighting coefficient that is close to infinity, capital is allocated to stocks that have a low rate of return so that the risk obtained is also low. The capital weight of stocks that have a large variance will tend to be smaller than stocks that have a small variance.
b. The optimal portfolio depends on the investor's preference for risk, as previously explained there are three types of investors:
    - Risk Seeker
      The optimal portfolio for investors who like to face risks (risk seekers) is a portfolio with a weighting coefficient $k = 0.01$. The sharia stock portfolio for risk seeker type investors consists of ADRO, INDF, PTPP, TLKM and UNTR stocks. Meanwhile, the non-Sharia stock portfolio for risk seeker type investors consists of BBCA, BBNI, BBRI, GGRM and PTBA stocks.
    - Risk Indifference
      The optimal portfolio for investors who are indifferent to risk (risk indifference) is a portfolio with a weighting coefficient of $1 \leq k \leq 100$. The sharia stock portfolio for risk indifference investors consists of ADRO, ASII, INDF, LSIP, PTPP, TLKM, UNTR and UNVR stocks. Meanwhile, the non-Sharia stock portfolio for risk indifference type investors consists of BBCA, BBNI, BBRI, GGRM, INTP, JSMR and PTBA stocks.
    - Risk Averse
      The optimal portfolio for investors who are risk averse is a portfolio with a weighting coefficient $k = 1000$. The sharia stock portfolio for risk averse investors consists of ADRO, ASII, INDF, LSIP, PTPP, TLKM, UNTR and UNVR stocks. Meanwhile, the non-shariah stock portfolio for risk averse investors consists of BBCA, BBRI, GGRM, INTP, JSMR and PTBA stocks.

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